Drawn & Engd by J. Scott.

BIBLIOTHECA HUNTERIANA
GLASGUENSIS.

Ag. 11. 5.

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BIBLIOTHECA HUNTERIANA
GLASGUENSIS.

Ag. 11. 5.

R. 8. 24

Of him, whose shape this Picture hath design'd;
Vertue, and learning, represent the Mind. W. S.

# THE

# Moſt Eaſy and Exact manner

## OF

## Reſolving all ſorts of Triangles, whether Plain or Sphericall, Rectangular or Obliquangular; with greater facility, then ever hitherto hath been practiſed.

Moſt neceſſary for all ſuch as would attaine to the exact knowledg of Fortification, Dyaling, Navigation, Surveying, Architecture, the Art of Shadowing, taking of Heights, and Diſtances, the uſe of both the Globes, Perſpective, the skil of making of Maps, the Theory of the Planets, the calculating of their motions, and of all other Aſtronomicall computations whatſoever.

Lately invented, and perfected, explained, commented on, and, with all poſſible brevity, and perſpicuity in the hiddeſt, and moſt re-ſearched myſteries, from the very firſt grounds of the Science it ſelfe, proved, and convincingly demonſtrated.

*By* T. U. *Student in the* Mathematick.

*LONDON*,

Printed for *William Hope*, at the ſign of the Unicorn neer the Royall Exchange, 1650.

TO THE
RIGHT HONOVRABLE,
And most noble
# LADY,
My deare and loving Mother, the Lady DOWAGER of *Cromartie*.

MADAM,

HIALL duty being the more binding in me, that I doe owe it to the best of *Mothers*; if in the discharge thereof I observe not the usuall manner of other *sonnes*, I am the lesse to blame, that their obligation is not so great as mine: Therefore in that I doe presume

preſume to imprint your Ladiſhips name in the Frontiſpiece of this Book, and proffer unto you a Dedication of that, which is beyond the capacity of other *Ladies*; my boldneſſe therein is the more excuſable, that in your perſon the moſt *vertuous woman* in the world is intreated to Patronize that, which by the *learnedeſt men* may happily be peruſed.

I am confident (*Madam*) that your gracious acceptance of this Preſent is the more eaſily obtainable, in that it is a grand-child of your own, whom I thus make tender of, to be ſheltered under the favor of your protection; and that unto your Ladiſhip it will not be the more unwelcome, for proceeding from the braines of him, whoſe *body* is not more yours by *generation*, then by a moſt equitable *purchaſe* are the *faculties* of his mind; the *dominion* which over my better *halfe* you, by your *goodneſſe*, have acquired, being in regard of my obedience, no leſſe *voluntary*, then that of the other is for procreation *naturall*.

Thus (*Madam*) unto you doe I totally belong, but ſo, as that thoſe exteriour parts of mine, which by birth are from your Ladiſhip derived, cannot be more fortunate in this their ſubjection (notwithſtanding the egregious advantages of bloud, and

conſan

conſanguinity thereby to them accruing) then my ſelfe am happy (as from my heart I doe acknowledge it) in the juſt right, your Ladiſhip hath to the eternall poſſeſsion of the never-dying powers of my ſoule. For, though (Soveraignty excepted) there be none in this Iſland more honourably deſcended then is your Ladiſhip, nor whoſe progenitors, theſe many ages paſt, have been (on either ſide) of a more Noble extraction: Yet, laying apart *Nobility*, *beauty*, *wealth*, *parentage*, and *friends*, which (together with many other gifts of fortune) have hitherto ſerved to adorn your Ladiſhip beyond others of your ſex, who for all theſe have been deſervedly renowned; and (in ſome meaſure) not eſteeming that properly to be yours, the receiving whereof did not altogether depend upon your owne election: it is the treaſure of thoſe excellent graces, wherewith inwardly you are enriched, that, in praiſing of your Ladiſhip is moſt to be pitch'd upon, and for the which you are moſt highly to be commended; ſeeing by the means of them, you, from your tendereſt yeeres upwards, untill this time, in the ſtate of both *Virginity* and *Matrimony*, have ſo conſtantly, and indefatigably proceeded in the courſe of *vertue*, with ſuch alacrity fixed your gallant *thoughts* on the ſweetneſſe thereof,

thereof; and thereunto ſo firmly and cheerefully devoted all your *words* and *actions*, as if *righteouſneſſe* in your Ladiſhip had been an inbred quality, and that in your *Will* there had beene no aptitude of declining from the way of reaſon.

This much is ſufficiently well known to thoſe, that have at any time enjoyed the honour of your Ladiſhips converſation, by whoſe moſt unpartiall reports, the Splendour of your reputation is both in this, and foraine Nations accounted precious, in the minds even of thoſe, that have never ſeen you. But in ſo much more eſpecially, doe the moſt judicious of either ſex admire the rare and ſublime *endowments*, wherewith your Ladiſhip is *qualified*, that (as a patterne of perfection, worthy to be univerſally followed) other Ladies (of what dignity ſoever) are truly by them eſteemed of the choiſer merit, the nearer they draw to the Paragon propoſed, and reſemble your Ladiſhip; for that, by vertue of your beloved ſociety, your neighbouring Counteſſes, and other greater Dames of your kindred and acquaintance, become the more illuſtrious in your imitation; amidſt whom, as *Cynthia* amongſt the obſcurer Planets, your Ladiſhip ſhines, and darteth the Angelick rayes of your matchleſſe example

on

on the ſpirits of thoſe, who by their good *Genius* have been brought into your favourable preſence to be enlightned by them.

Now (*Madam*) leſt, by inſiſting any longer upon this ſtraine, I ſhould ſeeme to offend that modeſty, and humility, which (without derogating from your heroick vertues) are ſeated in a conſiderable place of your ſoule, I will here, in all ſubmiſsion, moſt humbly take my leave of your Ladiſhip, and beſeech Almighty God, that it may pleaſe his Divine Majeſty ſo to bleſſe your Ladiſhip with continuance of dayes, that the ſonnes of thoſe whom I have not as yet begot, may attaine to the happineſſe of preſenting unto your Ladiſhip a brain-babe of more ſufficiencie and conſequence; and that your Ladiſhip may live with as much health, and proſperity, to accept thereof, and cheriſh it then, as (I hope) you doe now, at your vouchſafing to receive this, which (though diſproportionable, both to your Ladiſhips high deſerts, and to that fervencie of willingneſſe in me, ſometime to make offer of what is of better worth, and more ſutable to the grandour of your acceptance) in all ſincerity of heart (confiding in that candour and ingenuity, wherby your Ladiſhip is accuſtomed to value gifts,

according to the intention of the giver) and in all duty, and lowlineſſe of mind, together with my ſelfe in whole, and all my beſt endeavours, I tender unto your Ladiſhip, as becometh,

(*Madam*)

*Your Ladiſhips*

*moſt affectionate Sonne*

*and humble ſervant,*

THOMAS URQUHART.

# To the Reader.

TO write of Trigonometry, and not make mention of the illuſtrious Lord *Neper* of *Marchiſton*, the inventer of Logarithms, were to be unmindfull of him that is our daily Benefactor; theſe artificiall numbers by him firſt excogitated and perfected, being of ſuch incomparable uſe, that, by them, we may operate more in one day, and with leſſe danger of errour, then can be done without them in the ſpace of a whole week. A ſecret which would have beene ſo precious to Antiquity, that *Pythagoras*, all the ſeven wiſe men of *Greece*, *Archimedes*, *Socrates*, *Plato*, *Euclid*, and *Ariſtotle*, had (if coævals) joyntly adored him, and unanimouſly concurred to the deifying of the revealer of ſo great a Myſtery: and truly (beſides them) a great many other learned men, who for the laborioufneſſe of long and various Multiplications, Diviſions, and Radicall extractions of ſeverall ſorts, were deterred from the proſecuting, and divulging of their knowledge in the chiefeſt, and moſt noble parts of the Mathematicks, would have left behind them diverſe exquiſite Volumes, of an incomprehenſible value, if the Arithmetical equality of difference, agreeable to every continued Geometricall proportion, had been made known unto them.

Wherefore, I am infallibly perſwaded, that, in the eſtimation of ſcientifically diſpoſed ſpirits, the Philoſophers ſtone is but traſh to this invention, which will alwayes (in their judicious opinions) be accounted of more worth to the Mathematicall world, then was the finding out of *America*, to the King of *Spaine*; or the diſcovery of the near-

eſt way to the Eaſt-indies, would be to the Northerly occidentall Merchants.

What the merit then of the Author is, let the moſt envious judge: for my owne part, I doe not praiſe him ſo much, for that he is my Compatriot, as I muſt extoll the happineſſe of my Countrey, for having produced ſo brave a ſpark, in whom alone (I may with confidence averre, it is more glorious, then if it had beene the conquering Kingdome of five hundred potent Nations: for, by how much the gifts of the mind, are more excellent then thoſe of either body or fortune; by ſo much the divine effects of the faculties thereof, are of greater conſequence, then what is performed by meer force of Armes, or chance of Warre.

I might ſay more in commendation of this gallant man, but that my diſcourſe being directed to the Reader, he will poſſibly expect to be entertained with ſome other purpoſe then Encomiaſticks; and therefore, to undeceive him of thoſe hopes (if any ſuch there be) I will aſſure him, that to no other end I did require his obſervance here, but to be informed by me of the laudable endowments of that honourable Baron, whoſe eminencie above others (wher-ever he be ſpoke of) deſerveth ſuch an ample Elogie by it ſelfe, that the paper, graced with the receiving of his name and character, ſhould not be blurred with the courſe impreſſion of any other ſtuffe.

However the Reader ought not to conceive amiſſe for his being detained ſo long upon this Eulogiſtick ſubject, without the variety of any peculiar inſtruction beſtowed on him; ſeeing I am certaine there is nothing more advantagious to him, or that more efficaciouſly can tend to his improvement, then the imitation of that admirable Gentleman, whoſe immortall fame, in ſpite of time, will out-laſt all ages, and look eternity in the face.

*The Readers well-wiſher.* T. U.

# An Epænetick and Doxologetick Expreſſe, in commendation of this Book, and the Author thereof.

## To all Philomathets.

SEeing *Trigonometry*, which handſomly unlocketh the choyceſt, and moſt intime myſteries of the Mathematicks, hath beene hitherto expoſed to the World in a method, whoſe intricacy deterreth many from adventuring on it; We are all, and every one of us, by duty bound to acknowledge our ſelves beholding to the Author of this Treatiſe; who, by reducing all the ſecrets of that noble Science into a moſt exquiſite order, hath ſo facilitated the way to the Learner, that in ſeven weeks, at moſt, he may attaine to more knowledge therein, then otherwiſe he could doe for his heart in the ſpace of a twelve-moneth: And who, for the better encouragement of the ſtudious, hath ſo gentilely expatiated his ſpirits upon all its Actions, Principles, Analogies, Precepts, and whole ſubjected matter; that this Mathematicall Tractate doth no leſſe beſpeak him a good Poet, and good Orator, then by his elaboured Poems he hath ſhowne himſelfe already a good Philoſopher, and Mathematician.

Thus doth the various mixture of moſt excellent qualities

*ties in him, give ſuch evidence of the tranſcendent faculties of his mind, that, as the Muſes never yet inſpired ſublimer conceptions in a more refined ſtile, then is to be found in the accurate ſtrain of his moſt ingenious Epigrams: ſo, on the other part, are the abſtruſeſt difficulties of this Science by him ſo neatly unfolded, and with ſuch exactneſſe hath he reſolved the hardeſt, and moſt intangled doubts thereof; that, I may juſtly ſay, what praiſe (in his Epiſtle, or rather Preface, to the Reader) he hath beene pleaſed (out of his ingenuity, to confer on the learned, and honourable* Neper, *doth, without any diminution, in every jot, as duly belong unto himſelfe.*

*For, I am certainly perſwaded, he that uſeth Logarithms, ſhall not gaine ſo much time on the Worker by the naturall Sines and Tangents, as, by vertue of this ſuccinct manner of calculation, ſhall be got on him that knoweth it not, how compendiouſly ſoever elſe, with Addition and Subtraction, or Addition alone, he frame his Computations. However, he who, together with that of the Logarithms, maketh uſe of this invention, is in a way which will bring him ſo ſtraight and readily to the perfect practice of* Trigonometry, *that, compared with the old beaten path, trod upon by* Regiomontanus, Ptolomy, *and other ancient* Mathematicians, *it is like the Sea voyage, in regard of that by Land, betwixt the two Pillars of* Hercules *commonly called, the Straits of* Gibraltar) *whereof the one is but of ſix houres ſailing at moſt, and the other a journey of ſeven thouſand long miles.*

*If*

*If we then conſider how a great many, deſpairing ever to get out (if once entred) of the confuſed obſcurity wherein the doctrine of Triangles hath beene from time to time involved, have rather contented themſelves barely with Scale and Compaſſe, and other mechanick tooles and inſtruments, to proſecute their operations, and in any reaſonable meaſure to glance ſomewhat neare the truth, then, thorough ſo many peſterments, and harſh incumbrances, to touch it to a point, in its moſt indiviſible and infallible reality. And how others for all their being more induſtrious, in proving their Concluſions by the* Mediums *from which they are neceſſarily inferr'd, are nevertheleſſe (even when they have beſtowed half an age in the Trigonometricall practice) oftentimes ſo farre to ſeek, that, without a great deale of premeditation, adviſement, and recollecting of themſelves, they know not how to diſcuſſe ſome queries, corollaries, problems, conſectaries, proportions, wayes of perpendicular falling, and other ſuch like occurring debatable matters, incident to the ſcientifick meaſuring of Triangles; We cannot chooſe (theſe things being maturely perpended) but be much taken with the pregnancy of this device, whereby we ſhall ſooner hit to a minut upon the verity of an Angle or Side demanded, and trace it to the very ſource and originall, from whence it flowes, then another mechanically ſhall be able to come within three degrees thereof, although he cannot, for the ſame little he doth, afford any reaſon at all; And ſo ſuddenly reſolve any Trigonometricall queſtion (without*

*paines*

paines or labour, how perplexed ſoever it be) with all the dependances thereto belonging, as if it were a knowledge meerly infuſed from above, and revealed by the peculiar inſpiration of ſome favourable Angel.

Beſides theſe advantages, adminiſtred unto us by the meanes of this exquiſite Book, this maine commodity accreweth to the diligent Peruſers of it, that, inſtead of three quarters of a yeere, uſually by Profeſſors allowed to their Schollers for the right conceiving of this Science, which (notwithſtanding) through any little diſcontinuance, is by them ſo apt to be forgotten, that the expence of a week or two will hardly ſuffice to reſeat it in their memories; they ſhall not need, by this method, to beſtow above a moneth, and with ſuch eaſe and facility for retention, when they have learned and acquired it, that, if multiplicity of buſineſſes, or ſerious plodding upon other ſtudies happen to blot it out of their minds, they may as firmly recover in one quarter of an hour the whole knowledge and remembrance thereof, as when they had it beſt, and were moſt punctually verſed in it.

A ſecret (in my opinion) ſo precious, that (as the Author ſpoke of Marchiſton) I may with the like pertinencie avouch of him, that his Countrey and kindred would not have been more honoured by him, had he purchaſed millions of gold, and ſeverall rich territories of a great and vaſt extent; then for this ſubtile and divine invention, which will out-laſt the continuance of any inheritance, and remaine freſh in the underſtandings

*derſtandings of men of profound Literature, when houſes and poſſeſſions will change their owners, the wealthy become poor, and the children of the needy enjoy the treaſures of thoſe, whoſe heires are impoveriſhed.*

*Therefore, ſeeing for the many-fold uſes thereof in divers Arts and Sciences, in ſpeculation and practice, peace and war, ſport and earneſt, with the admirable furtherances we reape by it, in the knowledge of Sea and Land, and Heaven and Earth, it cannot be otherwiſe then permanent, together with the Authors fame, ſo long as any of thoſe endure; I will (God willing) in the ruines of all theſe, and when time it ſelfe is expired, in teſtimony of my thankfulneſſe in particular for ſo great a benefit, (if after the Reſurrection, there be any complementall affability) expreſſe my ſelfe then, as I doe now,*

The Authors

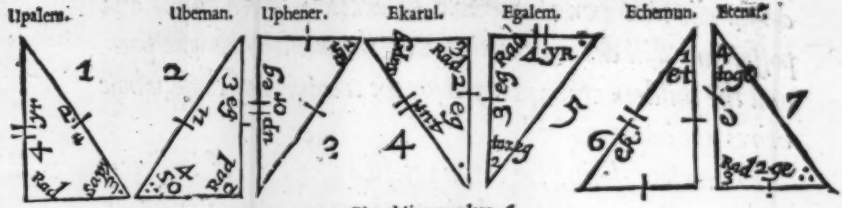
moſt affectionate,

and moſt humbly

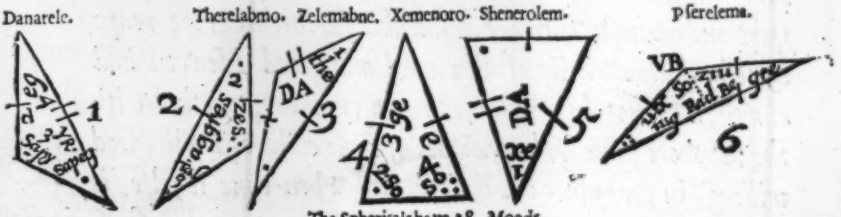
devoted ſervant

# The *Diatyposis* of the whole Doctrine of Triangles.

## The plane Triangles have 13. Moods. Planorectangulars 7.



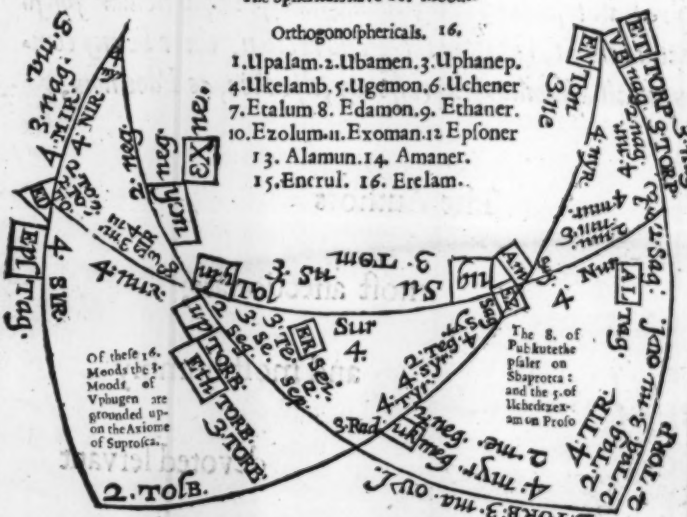

Planobliquangulars, 6.




The Sphericals have 28. Moods.

Orthogonosphericals. 16.

1. Upalam. 2. Ubamen. 3. Uphanep. 4. Ukelamb. 5. Ugemon. 6. Uchener 7. Etalum 8. Edamon. 9. Ethaner. 10. Ezolum. 11. Exoman. 12 Epsoner 13. Alamun. 14. Amaner. 15. Enerul. 16. Erelam.



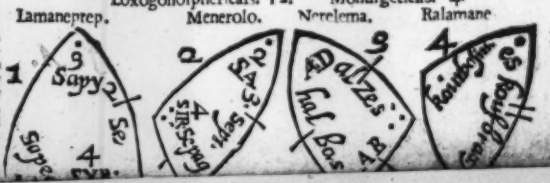
By these 16. Representatives, 1. Lec. 2. Ter. 3. Ruc 4. Cle. 5. Lu. 6. Tul. 7. Terc. 8. Tol. 9. Lec 10. Ar. 11. Tul. 12. Clet. 13. Cret. 14. Tar. 15. Tur. 16. Le (A. signifying an oblique Angle. E. the Perpendicular U. the subtendent C. Initiall, the complemēt of a side to a quadrant C. finall, the side continued to the Radius or a Quadrant. L. left. R. right. and T. one of the top Triangles of the Scheme) it is evidenced in what

Of these 16. Moods the 3. Moods of Vphugen are grounded upon the Axiome of Suprosca.

The 8. of Pubkutethe psaler on Sbaprotca: and the 5. of Uchedrzexam on Proso

part of the Diagram the Analogy of any of the 16. Moods begins, which being once knowne, the progressive sequence of the proportionable Sides & Angles is easily discerned out of the orderly involutions of the Figure it selfe. Here it is to be observed, that as the Book explaineth the Trissotetral Table so this Trigonodiatyposis unfoldeth all the intricate difficulties of the Book.

Loxogonosphericals. 12. Monurgeticks. 4.

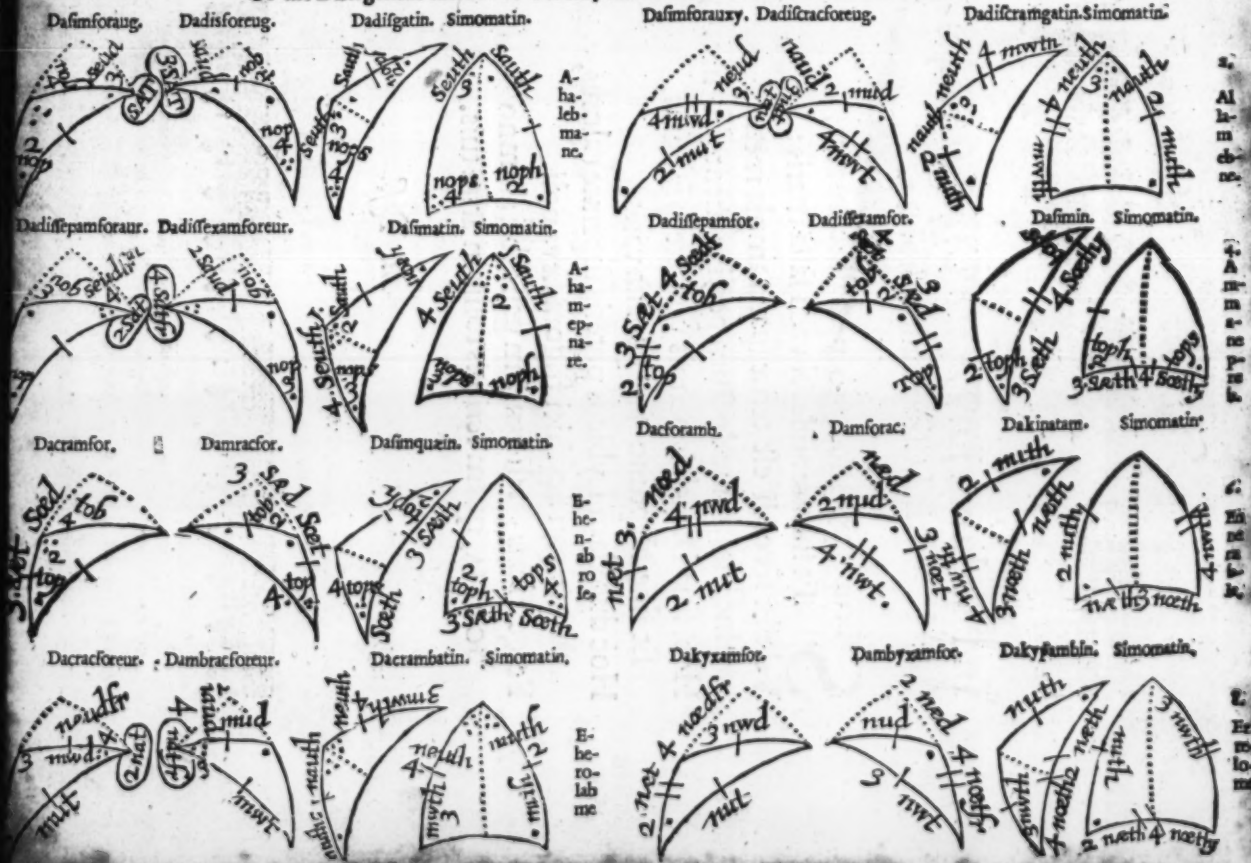



That the Schemes and Types of Triangulary Analogies are not seated in the roomes, where they are treated of, I purposely have done it, to the end, that being all perceived at one view, their multiplicity (which would appear confused in their [illegible]

dert besides that, this their being together in their due order, and rancked according to the exigence of the Sides or Angles, is such a furtherance to the memory, and illustration to the judgement, that it maketh Trigonometry, which of all Sciences [illegible]

Of the Disergeticks there be 8. Moods, each whereof is divided into foure Cases.

Of the Disergeticks there be 8. Moods, each whereof is divided into foure Cases.



## In Eruditum D. *Thomæ Vrquharti* equitis Triſſotetrados librum.

SI cupis ætherios tutò peragrare meatus,
Et ſulcare audes ſi vada ſalſa maris:
Vel tibi ſi cordi eſt terræ ſpatia ampla metiri,
Huc ades, hunc doctum percipe mente librum.
Hoc, ſine Dædaleis pennis volitare per auras,
Et ſine Neptuno nare per alta vales.
Hoc duce, jam Lybicos poteris ſuperare calores,
Atque pati Scythici frigora ſæva poli.
Perge *Thoma*; tali tandem gaudebit alumno
SCOTIA, quam ſcriptis tollis in aſtra tuis.

*Al. Roſs.*

# POSITIONS.

EVery Circle is divided into three hundred and ſixty parts, called Degrees, whereof each one is Sexageſimated, Subſexageſimated, Reſubſexageſimated, and Bireſubſexageſimated, in Minutes, Seconds, Thirds, Fourths, and ſo far forth as any Computiſt is pleaſed to proceed for the exactneſſe of a Reſearch, in the calculation of any Orbiculary Dimenſion.

2. As Degrees are the meaſure of Arches, ſo are they of Angles; but that thoſe are called Circumferentiall, theſe Angulary Degrees, each whereof is the three hundred and ſixtieth part of four right Angles, which are nothing elſe but the ſurface of a Plain to any point circumjacent; for any ſpace whatſoever about a point, is divided in 360. parts: And the better to conceive the Analogie that is betwixt theſe two ſorts of graduall Meaſures, we muſt know, that there is the ſame proportion of any Angle to 4. right Angles, as of an arch of ſo many circumferential degrees to the whole circumference.

3. Hence is it, that the ſame number ſerves the Angle, and the Arch that vaults it, and that divers quantities are meaſured (as it were) with the ſame graduall meaſure. Angles and Arches then are Analogicall, and the ſame reaſon is of both.

4. Seeing any given proportion may be found in numbers, and that any two quantities have the ſame proportion that the two numbers have, according to the which they are meaſured: if for the meaſuring of Triangles there muſt be certain proportions of all the parts of a Triangle, to one another known, and thoſe proportions explained in numbers it is moſt certain, all Magnitudes, being Figures at leaſt in power, and all Figures either Triangles, or Triangled, that the Arithmeticall Solution of any Geometricall queſtion, dependeth on the Doctrine of Triangles.

5. And though the proportion betwixt the parts of a Triangle cannot be without ſome errour; becauſe of crooked lines to right lines, and of crooked lines amongſt themſelves, the reaſon is inſcrutable, no man being able to finde out the exact proportion of the Diameter to the Circumference: yet both in plain Triangles, where the meaſure of the Angles is of a different ſpecies from the ſides, and in Sphericalls, wherein both the Angles and ſides are of a circular nature, crooked lines are in ſome meaſure reduced to right lines by the definition of quantity, which right lines, *viz.* Sines, Tangents,

and Secants, applyed to a Circle have in reſpect of the *Radius*, or half-Diameter.

6. And therefore, though the Circles Quadrature be not found out, it being in our power to make the Diameter, or the ſemi-Diameter, which is the *Radius* of as many parts as we pleaſe, and being ſure ſo much the more that the *Radius* be taken, the error will be the leſſer; for albeit the Sines, Tangents and Secants, be irrationall thereto for the moſt part, and their proportion inexplicable by any number whatſoever, whither whole or broken: yet if they be rightly made, they will be ſuch, as that in them all no number will be different from the truth by an integer, or unity of thoſe parts, whereof the *Radius* is taken: which is ſo exactly done by ſome, eſpecially by *Petiſcus*, who aſſumed a *Radius* of twenty ſix places, that according to his ſupputation (the Diameter of the Earth being known, and the Globe thereof ſuppoſed to be perfectly round) one ſhould not fail in the dimenſion of its whole Circuit, the nine hundreth thouſand ſcantling of the Million part of an Inch, and yet not be able, for all that, to meaſure it without amiſſe; for ſo indiviſible the truth of a thing is, that come you never ſo neer it, unleſſe you hit upon it juſt to a point, there is an errour ſtill.

## DEFINITIONS.

A Cord, or Subtenſe, is a right line, drawn from the one extremity to the other of an Arch.

2. A right Sine is the half Cord of the double Arch propoſed, and from one extremity of the Arch falleth perpendicularly on the *Radius*, paſſing by the other end thereof.

3. A Tangent is a right line, drawn from the Secant by one end of the Arch, perpendicularly on the extremity of the Diameter, paſſing by the other end of the ſaid Arch.

4. A Secant is the prolonged *Radius*, which paſſeth by the upper extremity of the Arch, till it meet with the fine Tangent of the ſaid Arch.

5. Complement is the difference betwixt the leſſer Arch, and a Quadrant, or betwixt a right Angle and an Acute.

6. The complement to a ſemi-Circle, is the difference betwixt the half-Circumference and any Arch leſſer, or betwixt two right Angles, and an Oblique Angle, whither blunt or ſharp.

7. The verſed ſine is the remainder of the *Radius*, the ſine

Complement

Complement being ſubtracted from it, and though great uſe may be made of the verſed ſines, for finding out of the Angles by the ſides, and ſides by the Angles: yet in Logarithmicall calculations they are altogether uſeleſſe, and therefore in my *Triſſotetras* there is no mention made of them.

8. In *Ambiguous ſphericalls*, which admit both of an Extrinſecall, and Intrinſecall demiſſion of the perpendicular, nineteen ſeverall parts are to be conſidered: *viz.* The Perpendicular, the Subtendentall, the Subtendentine, two Coſubtendents, the Baſall, the Baſidion, the chief Segment of the Baſe, two Cobaſes, the double Verticall, the Verticall, the Verticaline, two Coverticalls, the next Cathetoppoſite, the prime Cathetoppoſite, and the two Cocathetoppoſites: fourteen whereof, (to wit) the Subtendentall, the Subtendentine, the Coſubtendents, the Baſall, the Baſidion, the Cobaſes, the Verticall, the Verticaline, the Coverticalls, and Cocathetoppoſites, are called the firſt, either Subtendent, Baſe, Topangle, or Cocathetoppoſite, whither in the great Triangle or the little, or in the Correctangle, if they be ingredients of that Rectangular, whereof moſt parts are known, which parts are alwayes a Subtendent and a Cathetoppoſite: but if they be in the other Triangle, they are called the ſecond Subtendents, Baſes, and ſo forth.

9. The exteriall double Verticall is includ d by the Perpendicular, and Subtendentall, and divided by the Subtendentine: the interiall is included by coſubtendents, and divided by the Perpendicular.

## APODICTICKS.

THe Angles made by a right Line, falling on another right Line, are equall to two right Angles; becauſe every Angle being meaſured by an Arch, or part of a Circumference, and a right Angle by ninety Degrees, if upon the middle of the ground line, as Center, be deſcribed a ſemi-Circle, it will be the meaſure of the Angles, comprehended betwixt the falling, and ſuſtaining lines.

2. Hence it is, that the four oppoſite Angles made by one line, croſſing another, are always each to its own oppoſite equall; for if upon the point of Interſection, as Center, be deſcribed a Circle, every two of thoſe Angles will fill up the ſemi-Circle; therefore the firſt and ſecond will be equall to the ſecond and third, and conſequently the ſecond, which is the common Angle to both theſe couples being removed, the firſt will remain equall to the

third, and by the ſame reaſon, the ſecond to the fourth, which was to be demonſtrated.

3. If a right line falling upon two other right lines, make the alternat Angles equall, theſe lines muſt needs be Paralell; for if they did meet, the alternat Angles would not be equall; becauſe in all plain Triangles, the outward Angle is greater, then any of the remote inward Angles, which is proved by the firſt.

4. If one of the ſides of a Triangle be produced, the outward Angle is equall to both the inner, and oppoſite Angles together; becauſe according to the acclining or declining of the contermiall ſide, is left an Angulary ſpace, for the receiving of a paralell to the oppoſite ſide, in the point of whoſe occourſe at the baſe, the Exterior Angle is divided into two, which for their like, and alternat ſituation with the two Interior Angles, are equall each to its own conform to the nature of Angles, made by a right line croſſing divers paralells.

5. From hence we gather, that the three Angles of a plain Triangle, are equall to two rights; for the two inward, being equall to the Externall one, and there remaining of the three, but one, which was proved in the firſt *Apodictick*, to be the Externall Angles complement to two rights; it muſt needs fall forth (what are equall to a third, being equall amongſt themſelves) that the three Angles of a plain Triangle, are equall to two right Angles, the which we undertook to prove.

6. By the ſame reaſon, the two acute of a Rectangled plain Triangle, are equall to one right Angle, and any one of them, the others complement thereto.

7. In every Circle, an Angle from the Center, is two in the Limb, both of them having one part of the Circumference for baſe; for being an Externall Angle, and conſequently equall to both the Intrinſecall Angles, and therefore equall to one another; becauſe of their being ſubtended by equall baſes, *viz.* the ſemi-Diameters, it muſt needs be the double of the foreſaid Angle in the limb.

8. Triangles ſtanding between two paralells, upon one and the ſame baſe, are equall; for the Identity of the baſe, whereon they are ſeated, together with the Equidiſtance of the Lines, within the which they are confined, maketh them of ſuch a nature, that how long ſo ever the line paralell to the baſe be protracted, the Diagonall cutting of in one off the Triangles, as much of bredth, as it

gains of length, (the ones loſſe accruing to the profit of the other) Quantifies them both to an equality, the thing we did intend to prove.

9. Hence do we inferre, that Triangles betwixt two paralells, are in the ſame proportion with their baſes.

10. Therefore if in a Triangle, be drawn a paralell to any of the ſides, it divideth the other ſides, through which it paſſeth proportionally; for beſides that it maketh the four ſegments, to be four baſes, it becomes (if two Diagonall lines be extended from the ends thereof, to the ends of its paralell) a common baſe to two equall Triangles, to which two, the Triangle of the firſt two ſegments, having reference according to the difference of their baſes, and theſe two being equall, as it is to the one, ſo muſt it be to the other, and therefore the firſt baſe, muſt be to the ſecond, (which are the Segments of one ſide of the Triangle) as the third to the fourth, (which are the Segments of the ſecond) all which was to be demonſtrated.

11. From hence do we collect, that Equiangled Triangles have their ſides about the equall Angles proportionall to one another. *This ſayes* Petiſcus, *is the golden Foundation, and chief ground of Trigonometry.*

12. An Angle in a ſemi-Circle is right; becauſe it is equall to both the Angles at the baſe, which (by cutting the Diameter in two) is perceivable to any.

13. Of four proportionall lines, the Rectangled figure, made of the two extreames, is equall to the Rectangular, compoſed of the means; for as four and one, are equall to two and three, by an Arithmeticall proportion: and the fourth term Geometrically exceeding, or being leſſe then the third, as the ſecond is more, or leſſe then the firſt; what the fourth hath, or wanteth, from and above the third, is ſupplyed, or impaired by the Surpluſage, or deficiency of the firſt from and above the ſecond: Theſe Analogies being ſtill taken in a Geometricall way, make the oblong of the two middle, equall to that of the extreams, which was to be proved.

14. In all plain Rectangled Triangles, the Ambients are equall in power to the Subtendent; for by demitting from the right Angle a Perpendicular, there will ariſe two Correctangles, from whoſe Equiangularity with the great Rectangle, will proceed ſuch a proportion amongſt the Homologall ſides, of all the three, that if you ſet them right in the rule, beginning your Analogy at the main Subtendent,

tendent, (ſeeing the including ſides of the totall Rectangle, prove Subtendents in the partiall Correctangles, and the baſes of thoſe Rectanglets, the Segments of the great Subtendent) it will fall out, that as the main Subtendent is to his baſe, on either ſide (for either of the legs of a Rectangled Triangle, in reference to one another, is both baſe and Perpendicular) ſo the ſame baſes, which are Subtendents in the leſſer Rectangles, are to their baſes, the Segment, of the prime Subtendent: Then by the Golden rule we find, that the multiplying of the middle termes (which is nothing elſe, but the ſquaring of the comprehending ſides of the prime Rectangular) affords two products, equall to the oblongs made of the great Subtendent, and his reſpective Segments, the aggregat whereof by equation is the ſame with the ſquare of the chief Subtendent, or Hypotenuſa, which was to be demonſtrated.

15. In every totall ſquare, the ſupplements about the partiall, and Interior ſquares, are equall the one to the other; for by drawing a Diagonall line, the great ſquare being divided into two equall Triangles, becauſe of their ſtanding on equall baſes betwixt two paralells, by the ninth *Apoteſtick*, it is evident, that in either of theſe great Triangles, there being two partiall ones, equall to the two of the other, each to his own, by the ſame Reaſon of the ninth: If from equall things (*viz.* the totall Triangles) be taken equall things (to wit, the two pairs of partiall Triangles) equall things muſt needs remain, which are the foreſaid ſupplements, whoſe equality I undertook to prove,

16. If a right line cut into two equall parts be increaſed, the ſquare made of the additonall line, and one of the Biſegments, joyned in one, leſſe by the Square of the half of the line Biſected, is equall to the oblong contained under the prolonged line, and the line of Continuation; for if annexedly to the longeſt ſide of the propoſed oblong, be deſcribed the foreſaid Square, there will ſet out beyond the Quadrat-figure, a ſpace or Rectangle, which for being powered by the Biſegment and Additionall line, will be equall to the neereſt ſupplement, and conſequently to the other (the equality of ſupplements being proved by the laſt *Apoteſtick*) by vertue whereof, a Gnomon in the great Square, lacking nothing of its whole *Area*, but the ſpace of the ſquare of the Biſected line, is apparent to equalize the Parallelogram propoſed, which was to be demonſtrated.

17. From

17. From hence proceedeth this Sequell, that if from any point without a circle, two lines cutting it be protracted to the other extremity thereof, making two cords, the oblongs contained under the totall lines, and the exceſſe of the Subtenſes, are equall one to another; for whether any of the lines paſſe through the Center, or not, if the Subtenſes be Biſected, ſeeing all lines from the Center, fall Perpendicularly upon the Chordall point of Biſection (becauſe the two ſemi-Diameters, and Biſegments ſubſterned under equall Angles, in two Triangles evince the equality of the third Angle, to the third, by the fift *Apodictick*, which two Angles being made by the falling of one right line upon another, muſt needs be right by the tenth definition of the firſt of *Euchilde*) the *Bucarnon* of *Pythagoras*, demonſtrated in my fourteenth *Apodictick*, will by Quadroſubductions of Ambients, from one another, and their Quadrobiquadrequations, with the Hypotenuſa, together with other Analogies of equation with the powers of like Rectangular Triangles, comprehended within the ſame circle, manifeſt the equality of long Squares, or oblongs Radically meeting in an Exterior point, and made of the prolonged Subtenſes, and the lines of interception, betwixt the limb of the circle, and the point of concourſe, *quod probandum fuit.*

18. Now to look back on the eleaventh *Apodictick*, where according to *Petiſcus*, I ſaid that upon the mutuall proportion of the ſides of Equiangled Triangles, is founded the whole Science of *Trigonometry*, I do here reſpeak it, and with confidence maintain the truth thereof: becauſe, beſides many others, it is the ground of theſe Subſequent Theorems: 1. The right ſine of an Arch, is to its co-ſine, as the *Radius* to the co-tangent of the ſaid Arch. 2. The co-ſine of an Arch, is to its ſine, as the *Radius* to the Tangent of the ſaid Arch. 3. The Sines, and co-Secants: the Secants, and co-Sines: and the Tangents, and co-Tangents, are reciprocally proportionall. 4. The *Radius* is a mean proportionall, betwixt the Sine, and co-Secant: the Secant, and co-Sine: and the Tangent, and co-Tangent: The verity of all theſe (If a Quadrant be deſcribed, and upon the two *Radiuſes* two Tangents, and two or three Sines be erected (which in reſpect of other Arches will be co-Sines and co-Tangents) and two Secants drawn (which are likewiſe co-Secants) from the Center to the top of the Tangents) will appear by the foreſaid reaſons, out of my eleaventh *Apodictick*.

The

# The Trissotetras.

Plain. Sphericall.

## Plain Trissotetras.

### Axiomes four.

| | | | |
|---|---|---|---|
| 1. *Rulerst* | *Uradesso:* | *Directory:* | *Enodandas.* |
| | *Eradetul:* | *Uphechet:* | 3. *Orth.* 1. *Obl.* |
| 2. *Eproso* | *Directorie:* | *Enodandas* | 3. *Ax. Grediftal: Dir.* ⊕. |
| | *Pubkegdaxesh:* | 4. *Orth.* 3. *Obl.* | 4. *Ax. Bagrediffiu: Dir.* ⚓. |

# The Planorectangular Table:

### Figures four.

| | | | |
|---|---|---|---|
| 1. *Va*le* | *Datas. Quæsitas.* | Resolvers. | |
| | *Vp* Al§em.* | *Rad--V--Sapy☞Yr.* | |
| 2. *Ve*mane* | *Vb*em§an.* | *V--Rad--Eg☞So.* | |
| | | Præsubserv. | Possubserv. |
| | *Vph*en§er.* | *Vb*em§an.* | *Up*al§em, or, Eg*al§em.* |
| 3. *Ena*ve* | *Ek*ar§ul.* | *Sapeg--Eg--Rad☞Ur.* | |
| | *Eg*al§em.* | *Rad--Taxeg--Eg☞Yr.* | |
| | | Præsubserv. | Possubserv. |
| 4. *Ere*va* | *Ech*em§un.* | *Et*en§ar.* | *Ek*ar§ul.* |
| | *Et*en§ar.* | *E--Ge--Rad☞Toge.* | |

# The Planobliquangular Table:

### Figures four.

| | | | |
|---|---|---|---|
| 1. *Alahe*mo* | *Da*na*re§le.* | *Sapeg---Eg---Sapyr☞Yr.* | |
| 2. *Emena*role* | *The*re*lab§mo.* | *Aggres--Zes--Talfagros☞Talzo.* | |
| | | Præsubserv. | Possubserv. |
| | *Ze*le*mab§ne.* | *The*re*lab§mo.* | *Da*na*re§le.* |
| 3. *Enero*lome* | *Xe*me*no§ro.* | *E--So--Ge☞So.* | |
| | | Præsubserv. | Possubserv. |
| | *She*ne*ro§lem.* | *Xe*me*no§ro.* | *Da*na*re§le* |
| | | Præsubserv. | Possubserv. |
| 4. *Erele*a* | *Pse*re*le§ma.* | *Bagreziu.* | *Vb*em§an.* |
| Finall Resolver. | | *Uxiug--Rad--Iug--☞Sor.* | |

The

# The Sphericall Triſſotetras.

## Axiomes three.

1. *Suproſca.* 2. *Sbaprotca.* 3. *Seproſo.*
*Dir. uphugen. pubkutethepſaler. uchedezexam.*

## The Orthogonoſpherical Table.

Figures 6. Datoquæres 16.

| | Dat. Quæſ. | Reſolvers. |
|---|---|---|
| 1. *Ualam*menep* | *Up*al§am.* | *Torb—Tag—Nu ☞ Mir.* |
| | *Ub*am§en.* | *Nag—Mu—Torp ☞ Myr.* or, |
| | | *Torp—Mu—Lag ☞ Myr.* |
| | *Uph*an§ep.* | *Tol—Sag—Su ☞ Syr.* |
| 2. *Veman*nore* | *Uk*el§amb.* | *Meg—Torp—Mu ☞ Nir.* or, |
| | | *Torp—Teg—Mu ☞ Nir.* |
| | *Ug*em§on.* | *Su—Seg—Tom ☞ Sir.* or, |
| | | *Tom—Seg—Ru ☞ Sir.* |
| | *Uch*en§er.* | *Neg—To—Nu ☞ Nyr.* or, |
| | | *To—Le—Nu ☞ Nyr.* |
| 3. *Enar*rulome* | *Et*al§um.* | *Torp—Me—Nag ☞ Mur.* |
| | *Ed*am§en.* | *To—Neg—Sa ☞ Nir.* |
| | *Eth*an§er.* | *Torb—Tag—Se ☞ Tyr.* |
| 4. *Erol*lumane* | *Ez*ol§um.* | *Sag—Sep—Rad ☞ Sur.* or, |
| | | *Rad—Seg—Rag ☞ Sur.* |
| | *Ex*om§an.* | *Ne—To—Nag ☞ Sir.* or, |
| | | *To—Le—Nag ☞ Sir.* |
| | *Epſ*en§er.* | *Tag—Tolb—Te ☞ Syr.* or, |
| | | *Tolb—Mag—Te ☞ Syr.* |
| 5. *Acha*ve* | *Al*am§un.* | *Tag—Torb—Ma ☞ Nur.* or, |
| | | *Torb—Mag—Ma ☞ Nur.* |
| | *Am*an§er.* | *Say—Nag—To ☞ Nyr.* or, |
| | | *To—Noy—Ray ☞ Nyr.* |
| 6. *Eſhe*va* | *En*er§ul.* | *Ton—Neg—Ne ☞ Nur.* |
| | *Er*el§am.* | *Sei—Teg—Torb ☞ Tir.* or, |
| | | *Torb—Tepi—Rexi ☞ Tir.* |

# The Loxogonoſpherical Triſſotetras.

Monurgetick Diſergetick.

## The Monurgetick Loxogonoſpherical Table.

Axiomes two.

1. *Seproſo. Dir. Lame.* 2. *Parſes. Dir. Nera.*

Figures two. Moods four.

| Figures. | Datas. | Quæſ. | Reſolvers. |
|---|---|---|---|
| 1. *Datamiſta* | *Lam*an*ep§rep.* | *Sapeg—Se—Sapy* | *Syr.* |
| | *Me*ne*roſo.* | *Sepag—Sa—Sepi* | *Sir.* |
| 2. *Datapura* | *Ne*re*le§ma.* | *Hal Baſalduleg* | *Sad Sab Re Regal ſBis*ir.* |
| | *Parſes—Powto—Parſadſab* | | *Powſalvertiſ R* |
| | *Ra*la*ma§ne.* | *Kour Bfaſines (ereled)* | *Kouf Br*axypopyx.* |

## The Loxogonoſpherical Diſergeticks

Axiomes foure.

1. *Na Badproſver.* 2. *Naverpror Tes.* 3. *Siubpror Tab.* 4. *Niubprodeſver.*
*Dir. Alama. Allera. Ammena. Errenna.*

Figures 4. Moods 8.

Fig. M. Sub Ref. Dat. Præm. Cathetotheſis. Final Reſolvers.

*Cafrugpiq.*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| *Ab* | *A* | | | | | | | | |
| | *La* | *Vp* | *Tag* | *ut *Op* | *§At* | *Daſimforaug* | *Sat-nop-Seud* | † | *nob. Kir.* |
| *A* | *Meb* | *Al* | *Nu* | *ud *Ob* | *§Aud* | *Dadisforeug* | *Saud nob-Sat* | † | *nop. Ir.* |
| | *Na* | *Am* | *Mir* | *utb*Opb* | *§Autb* | *Dadiſgatin* | *Sautb-nopb-Seutb* | † | *nops Ir.* |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 2. *Leb* | 2. | Sub. | Ref. | Dat. | Præm. | Cathetotheſis. | Final Reſolvers. | | |
| | *Al* | | | | | *Cafyxegeq.* | | | |
| *Ma* | *La* | *Vp* | *Tag* | *ut *op* | *§at* | *daſimforauxy* | *nat -mut -n:ud* | † | *mwd* |
| | *Meb* | *Al* | *Nu* | *ud *ob* | *§aud* | *dadiſcracforeug* | *naud -mud -nat* | † | *mwt* |
| *Ne* | *Ne* | *Am* | *Mir* | *utb*opb* | *§autb* | *dadiſcramgatin* | *nautb-mutb-neutb* | † | *mwth* |

Fig.

Fig. M. *Cathetotheſis. Plus minus.*

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| A | A | Sub. | Re. | Dat. | Pr. | Caſtiq. | | Final | Reſolvers. | Sindifora. |
| | | | | | | | | | | At |
| | Ma | up | Tag | ut*Op | § At | Dadiſſepamfor aur | Nop-Sat-Nob | ☞ | Seudfr | Autir. |
| Ha | Nep | Al | Nu | ud*Ob | § Aud | Dadiſſexamfor eur | Nob-Saud-Nop | ☞ | Saifr | Eutir. |
| | | | | | | | | | | Aud |
| | Ra | Am | Mir | utb*Opb | § Auth | Daſimatin | Noph-Sauth-Nops | ☞ | Seuthj | Authir. |

*Cathetotheſis. Plus minus.*

Mep 4.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Am | Sub. | Re. | Dat. | Pr. | Cafregpagiq. | | Final | Reſolvers | Sindiſoriu. |
| | | | | | | | | | | Æt |
| Na | Ma | ub | Mu | ut*Op | § Æt | Dadiſſepamfor | Tob-Top-Sæt | ☞ | Sædfr | Dyr. |
| | Nep | Am | Lag | ud*Ob | § Æd | Dadiſſexamfor | Top-Tob-Sæd | ☞ | Sætfr | Dyr. |
| | | | | | | | | | | Æd |
| Re | Reb | En | Myr | utb*Oph | § æth | Daſimin | Tops-Toph-SÆth | ☞ | Sæthj æth | Syr. |

Fig. M. *Cathetotheſis.*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Eb | En | Sub. | Re. | Dat. | Pr. | Cafregpigeq. | Final | Reſolvers. | |
| | Er | ub | Mu | ut*Op | § æt | Dacramfor | Sæd-Top-Sæt | ☞ | Tob. Kir. |
| En | Ab | Am | Lag | ud*Ob | § æd | Damracfor | Sæt-Tob-Sæd | ☞ | Top. Ir. |
| | Lo | En | Myr | utb*Oph | § æth | Daſimquein | Sæth-Toph-Sæth | ☞ | Tops. Ir. |

*Cathetotheſis.*

Ab 6.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | En | Sub. | Re. | Dat. | Q. Pr. | Cafregpiq. | Final | Reſolvers. | |
| Ro | Ne | ub | Mu | ut*Op | § æt | Dacforamb | Næt-Nut-Næd | ☞ | Nwd. Yr. |
| | Rab | Am | Lag | ud*Ob | § æd | Damforac | Næd-Nud-Næt | ☞ | Nwt. Yr. |
| Le | Le | En | Myr | utb*Oph | § æth | Dakinatam | Næth-Nuth-Næth | ☞ | Nwth. Yr. |

Fig. M. *Cathteotheſis. Plus minus.*

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Eb | E | Sub. | Re. | Dat. | Pr. | Caſtiq. | | Final | Reſolvers. | Sindifora. |
| | | | | | | | | | | At |
| | Re | up | Tag | ut*Op | § at | Dacracforaur | Mut-Nat-Mwd | ☞ | Neudfr | Autir. |
| Er | Lo | Al | Nu | ud*Ob | § aud | Dambracforeur | Mud-Naud-Mwt | ☞ | Natfr | Autir. |
| | | | | | | | | | | Aud |
| | Mab | Am | Mir | utb*Oph | § auth | Dacrambatin | Muth-Nauth-Mwth | ☞ | Neuthj | Authir |

*Cathetotheſis. Plus minus.*

Om 8.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Er | Sub. | Re. | Dat. | Pr. | Cacurgyq. | Final | Reſolvers. | | Sindiſoriu. |
| | | | | | | | | | | Æt |
| Ab | Re | ub | Mu | ut*Op | § æt | Dakyxamfor | Nut-Nat-Nwd | ☞ | Nædfr | Dyr. |
| | Lo | Am | Lag | ud*Ob | § æd | Dambyxamfor | Nud-Nad-Nwt | ☞ | Nætfr | Dyr. |
| | | | | | | | | | | Æd |
| Me | Me | En | Myr | utb*Oph | § æth | Dakypambin | Nuth-Nath-Nwth | ☞ | Næthj | Æth Syr. |

THe noveky of theſe words I know will ſeeme ſtrange to ſome, and to the eares of illiterate hearers ſound like termes of Conjuration: yet ſeeing that ſince the very infancie of learning, ſuch inventions have beene made uſe of, and new words coyned, that the knowledge of ſeverall things repreſentatively confined within a narrow compaſſe, might the more eaſily be retained in a memory ſuſceptible of their impreſſion (as is apparent by the names of *Barbara, Celarent, Darii, Ferio,* and fifteen more Syllogiſtick Moods, and by thoſe likewiſe of *Gammuth, A-re, Bmi, C-fa-uth,* and ſeventeen other ſteps of *Guidos Scale,* which are univerſally received by men of underſtanding, and that have their ſpirits tuned to the harmony of reaſon) I know not why Logick and Muſick ſhould be rather fitted with ſuch helps then Trigonometrie, which, for certitude of demonſtration, hath been held inferior to no ſcience, and for ſublimity and variety of object, is the primeſt of the Mathematicks. This is the cauſe why I framed the *Triſſotetras,* wherein the termes by me invented, without regard of the initiall letters of the words by them expreſſed, are compoſed of ſuch as, joyned together, are of moſt eaſie pronunciation; as the Tangent complement of a Subtendent is ſooner uttered by *Mu* then by T C S; and the Secant complement of the ſide required, by *Ry,* then (in the uſuall apocopating way) by the firſt ſyllables or letters of Secant complement, ſide, and required; and conſidering that without opening of the mouth no word can be ſpoken, which overture is performed by the vowel, to all the ſides and Angles I deſigned vowels, that in the coaleſcencie of ſyllables, Sines, Tangents, and Secants might the better conſound therewith.

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## The explanation of the *Triſſotetras.*

*A.* ſignifieth an Angle: *Ab.* in the Reſolvers ſignifieth *abſtraction,* but in the Figures and Datoquæres the *Angle between*: *Ac.* or *Ak.* the *acute Angle*. *Ad.* Addition. *Æ.* the firſt baſe: *Amb.* or *Am.* an obtuſe Angle: *As* Angles in the plurall number. *At.* the double verticall, *whether externall or internall. Au.* the firſt vertical Angle: *Ay,* the Angle adjoyning to the ſide required.

*B.*

*B.* or *Ba.* the true baſe: *Bis* the double of a thing.

*Ca.* the perpendicular: *Cra.* the concurſe of a given and required ſide: *Cur.* the concurſe of two given ſides.

*D.* the partiall or little rectangle or rectanglet. *Da.* the datas. *Di.* or *Dif.* the difference: *Dir.* the directories. *D. q.* Datoquæres. *Diſſ.* of unlike natures.

*E.* a ſide: *Eb.* the ſide between: *Enod*, enodandas: *Ereled.* turned into ſides: *Es*, ſides in the plurall number: *Ei*, the ſide conterminall with the Angle required: *Eu*, the ſecond verticall Angle.

*F.* the new baſe, or angularie baſe, it being an Angle converted into a ſide: *Fig.* figures: *Fin. Reſ.* finall reſolvers: *For*, or *Fo*, outwardly, often made uſe of in the *Cathetotheſis*: *Fr.* a ſubducting of a leſſer from a greater, whether it be Side or Angle.

*G.* An Angle or Side given: *Gre*, or *aggre*, the ſumme or aggregat.

*Hal*, or *Al*, the halfe.

*I. Vowel*, an Angle required: *J Conſonant*, the addition of one thing to another uſed in the clauſuls of ſome of the finall Reſolvers. *In*, intus or inwardly, and ſometimes turned into *Iu*, the ſegments of the baſe, or the ſegmented baſe.

*K.* The complement of an Angle to a Semicircle.

*L.* The Secant: *Leg.* one of the comprehending ſides of an Angle. This repreſentative is once only mentioned.

*M.* A Tangent complement.

*N.* A Sine complement.

*O.* An oppoſite Angle, or rather Cathetoppoſite: *Ob.* the next cathetoppoſite Angle, by ſome called the firſt oppoſite: *Op.* the prime cathetoppoſite Angle, by ſome called the ſecond oppoſite. *Oph*, the firſt of the coopposite Angles: *Orth*, an acute Angle: *Ops*, the ſecond of the coopposits: *Os*, oppoſite Angles in the plurall number. *Oe*, the ſecond baſe; *Ou*, the Angle oppoſite to the baſe.

*P.* Oppoſite, whether Angle or ſide: *Par.* a parallelogram or oblong. *Præſ.* præſubſervient: *Poſſub.* poſſubſervient: *Pro.* proportionall: *Prod.* directly proportionall: *Pror*, reciprocally proportionall: *Pow.* the Square of a Line: *Præn.* prænoſcendas.

*Q.* Contin-

*Q.* Continued if need be. *Quæſ. Quæſitas. Quæ. Quære,* or *Required.*

*R.* The Secant complement, and ſometimes in the middle of the Cathetothesis ſignifies *required,* as always in the latter end of a *finall reſolver* it doth by way of emphaſis, when it followes *I.* or *Y.* *R.* likewiſe in the Axiom of *Ruleriſt* ſtands for *Radius.*

*Rq.* the Radius, and in the Scheme the middle angularie Radius.

*S.* The Sine, and in the cloſe of ſome Reſolvers, the *Summe. Sim.* of like affection or nature: *Subſ.* Subſervient.

*T.* The Tangent. *To.* the Radius or total Sine, but in the *Diagram* it is taken for the left angularie Radius: *Tw.* the right angularie Radius in the *Scheme* propoſed: *Tol.* the firſt hypotenuſal Radius thereof. *Tom.* the ſecond hyp. Radius. *Ton.* the third hyp. Rad. *Tor.* the fourth hyp. Rad. *Tolb.* the baſiradius on the left hand. *Torb.* the baſiradius on the right. *Tolp.* the Cathetorabdos, or Radius on the left. *Torp.* the Cathetoradius on the right. *Th.* the correctangle.

*U.* The Subtendent ſide. *V.* conſonant, to avoid vaſtneſſe of gaping, expreſſeth the ſame in ſeverall figures. *Ur.* the Subtendent required.

*W.* The ſecond Subtendent.

*X.* Adjacent or Conterminal.

*Y.* The ſide required.

*Z.* The difference of Segments, and is the ſame with *di,* or *dif.*

*Nevertheleſſe the* Reader *may be pleaſed to obſerve, that no Conſonants in the* Figures *or* Moods *are repreſentative ſave* P. *and* B. *and that only in a few; both theſe two and all the other Conſonants meerly ſerving to expreſſe the order and ſeries of the* Moods *and* Figures *reſpectively amongſt themſelves, and of their conſtitutive parts in regard of one another.*

# ANIMADVERSIONS.

IN the letter *T.* I have been ſomething large in the enumeration of ſeverall Radiuſes; for there being eleven made uſe of in the grand *Scheme,* whereof eight are Circumferentiall, and three Angularie, that they might be the better diſtinguiſhed from

from one another, when falling in proportion we ſhould have occaſion to expreſſe them; I thought good to allot to every one of them its owne peculiar Character: all which I have done with the more exactneſſe, that by the variety of the Radiuſes amongſt themſelves, when any one of them in particular is pitched upon, we may the ſooner know what part of the *Diagram*, by meanes thereof, is fitteſt for the reſolving of any Orthogonoſphericall problem: though indeed, I muſt confeſſe, when ſometimes to a queſtion propounded, I adapt a figure apart, I doe indifferently (excluding all other characters) make uſe of *To*, or *Rad*, or *R* onely for the totall Sine, which, without any obſcurity or confuſion at all, I have practiſed for brevities ſake.

Likewiſe, it being my maine deſigne in the framing of this Table, to make all capable trigonometrically-affected Students with much facility and litle labour attaine to the whole knowledge of the noble Science of the doctrine of Triangles, I deemed it expedient, the more firmely and readily to imprint the ſeverall *Datoquares* or præſcinded Problems thereof in their memories, to accommodate them accordingly with letters proper for the purpoſe; which, if the ingenious Reader will be pleaſed to conſider, he will find, by the very letters themſelves, the place and number of each *Datoquare*. This is the reaſon why my *Triſſotetras* (conforme to the Etymologie of its name) is in ſo many divers Ternaries, and Quaternaries divided; and that the ſharp, meane, blunt, double, and Liquid Conſonants of the Greek Alphabet, are ſo orderly beſtowed in their ſeverall roomes, being all and every one of them ſeated according to the nature of the Moods and Figures, whoſe characteriſticks they are.

Thirdly, the Moods of the Planotriangular Table, being in all thirteene, whereof there be ſeven Rectangular, and ſix Obliquangular, are fitly comprehended by the three blunt, three meane, three ſharp, and foure double Conſonants, the Hebrew Shin being accounted for one of them.

Fourthly, the ſixteen Moods of the Orthogonoſphericall Triſſotetras are contained under three ſharp, three mean, three blunt, three double, and foure Liquids, which foure doe orderly particulariſe the Binaries of the laſt two Figures.

Fifthly, the foure Monurgetick Loxogonoſphericals are deciphred

phred by each its owne Liquid in front, according to their literall order.

Sixthly, the eight Loxogonoſpherical Diſergeticks are alſo diſtinguiſhed by the foure Liquids, but with this difference from the Monurgeticks, that the Vowels of *A* and *E* precede them in the firſt ſyllable, importing thereby the Datas of an *Angle* or a *Side*. Now becauſe theſe Diſergeticks are eight in all, there being allotted to every Liquid that characteriſeth the Figures, the better to diverſifie the firſt and ſecond Datas of each reſpective binarie from one another, (in ſo farre as they have referenee to each its own Quæſitum) the Figurative Liquid is doubled when a Side is required, and remaineth ſingle when an Angle.

a Binary

Furthermore, in the Oblique Sphericodiſergeticks, ſo farre as the ſenſe of the Reſolvers could beare it, I did trinifie them with letters convenient for the purpoſe, according to the ſeverall caſes of their Datoquæres, whoſe diverſity reacheth not above the extent of $\pi.\ \mathcal{C}.\ \varphi.$ and $\tau.\ \delta.\ \beta.$

I had almoſt omitted to tell you, that for the more variety in the laſt two Figures of the Orthogonoſphericals are ſet downe the two letters of *Ch.* and *Shin*, the firſt a Spaniſh, and the ſecond an Hebrew letter. Now if to thoſe helps for the memorie which in this Table I have afforded the Reader, both by the Alphabetical order of ſome Conſonants, and homogeneity of others in their affections of ſharpneſſe, meanneſſe, obtuſity, and duplicity, he joyne that artificiall aid in having every part of the *Chemo* locally in his mind (of all wayes both for facility in remembring, and ſtedfaſtneſſe of retention, without doubt, the moſt expedite) or otherwiſe place the repreſentatives of words, according to the method of the Art of memory, in the ſeverall corners of a houſe (which, in regard of their paucity are containable within a Parlour or dining roome at moſt) he may with eaſe get them all by heart in leſſe then the ſpace of an houre: *which is no great expence of time, though beſtowed on matters of meaner conſequence.*

# The Commentary.

THe *Axiomes* of plain Triangles are foure, *viz. Rulerſt, Eproſo, Grediſtal*, and *Bagredaſſiu.*

*Rulerſt*, that is to ſay, the Subtendent in plain Triangles may be either Radius or Secant, and the Ambients either Radius, Sines, or Tangents; for it is a maxime in Planangular Triangles, that any ſide may be put for Radius, grounded on this, that from any point at any diſtance a Circle may be deſcribed: therefore if any of the ſides of a plain Triangle be given together with the Angles, each of the other two ſides is given by a threefold proportion, that is, whether you put that, or this, or the third ſide for the Radius; which difference occaſioneth both in plaine and Spherical Triangles great variety in their calculations.

The Branches of this Axiom are *Vradeſſo* and *Eradetul.*

*Vradeſſo*, that is when the Hypotenuſa is Radius, the ſides are Sines of their oppoſit Angles; ſo that there be two Arches deſcribed with that Hypotenuſal identity of diſtance, whoſe Centers are in the two extremities of the Subtendent; for ſo the caſe will be made plaine in both the Legs, which otherwiſe would not appeare but in one.

*Eradetul*, when any of the ſides is Radius, the other of them is a Tangent, and the Subtendent a Secant. The reaſon of this is found in the very definitions of the Sines, Tangents, and Secants, to the which, if the Reader pleaſe, he may have recourſe; for I have ſet them downe amongſt my Definitions. Hence it is (according to Mr. *Speidels* obſervation in his book of Spherics) that the Sine of any Arch being Radius, that which was the totall Sine becomes the Secant complement of the ſaid Arch, and that the Tangent of any Arch being Radius, what was Radius becomes Tangent complement of that Arch.

The Directory of this Axiome is *Vphechet*, which ſheweth us, that there be three Planorectangular Enodandas belonging thereto, *viz. Vphener, Echemun*, and *Etenar*; as for *Pferelema*, which is the Loxogonian one pointed at in my Triſſotetras, becauſe it is but a partiall Enodandum, I have purpoſely omitted to mention it in the Directory of *Eradetul.*

The ſecond Axiome is *Eproſo*, that is, the ſides are proportionall to one another as the Sines of their oppoſite Angles; for ſee-

ing about any Triangle a Circle may be circumſcribed, in which caſe each ſide is a cord or Subtenſe, the halfe whereof is the Sine of its oppoſite Angle, and there being alwayes the ſame reaſon of the whole to the whole, as of the halfe to the halfe, the ſides muſt needs be proportionall to one another, as the Sines of their oppoſite Angles, *quod probandum erat.*

The Directory of this ſecond Axiome is *Pubkegdaxeſh*, which declareth that there are ſeven Enodandas grounded on it, to wit, foure Rectangular, *Upalem, Ubeman, Ekarul, Egalem*, and three Obliquangular, *Danarele, Xemenoro*, and *Shenerolem.*

The third Axiom is *Grediſtal*, that is, in all plain Triangles, As the ſumme of the two ſides is to their difference, ſo is the Tangent of the halfe ſum of the oppoſite Angles to the Tangent of half their difference; for if a Line be drawne equall to the ſumme of the two ſides, and if on the point of Extenſion with the diſtance of the ſhorter ſide a Semicirele be deſcribed, and that from the extremity of the protracted Line a Diameter be drawne thorough the Circle where it toucheth the top of the Triangle in queſtion, till it occurre with a parallel to the third ſide, there will ariſe two Equicrurall Triangles, one whereof having one Angle common with the Triangle propoſed, and the three of the one being equall to the three of the other, any one of the equall Angles in the foreſaid *Iſoſceles* muſt needs be the one halfe of the two unknowne Angles. This is the firſt ſtep to the obtaining of what we demand. Then do we find that the third ſide cutteth the ſides of the greateſt Triangle according to the Analogie required, which is perceivable enough, if with the diſtance of the outmoſt Parallel from the lower end thereof as Center, be deſcribed a new Circle; for then will the Tangents be perſpicuous, and ſo much the more for their Rectangularity, the one with the Radius, and the other with its Parallel, which, being touched at an Angle deſcribed in a Semicircle, confirmeth the Rectangularity of both. By the Parallels likewiſe is inferred the equality of the alternate Angles, whoſe addition and ſubduction to and from halfe the ſum of the two unknown Angles make up both the greater and leſſer Angle. *Hereby it is evident how the ſum of the two ſides, &c. which was to be proved.*

The Directory of this third *Axiom* is 8. onely; for it hath no Enodandum but *Therelabmo.*

The

The fourth *Axiom* is *Bagrediſſin*, that is, As the Baſe or greateſt ſide is to the ſumme of the other ſides, ſo the difference of the other ſides to the difference of the Segments of the Baſe; for if upon the Center of the verticall Angle with the diſtance of the ſhorteſt ſide be deſcribed a Circle, it will ſo cut the two greater ſides of the given Triangle, that, finding thereby two Oblongs of the nature of thoſe whoſe equality is demonſtrated in my *Apodicticks*, we may inferre (the Oblong made of the ſumme of the ſides, and difference of the ſides being equall to the Oblong made of the Baſe, and the difference of its Segments) that their ſides are reciprocally proportionall; that is, As the greateſt ſide is to the ſum of the other ſides: ſo the difference of the other ſides, to the difference of the Segments of the Baſe, or greater ſide.

The *Directory* of this *Axiom* is ⊥. and its onely Enodandum, (though but a partiall one) *Pſerelema*.

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# The Planorectangular Table hath foure Figures.

IT is to be obſerved, that *Figure* here is not taken Geometrically, but in the ſenſe that it is uſed in the Logicks, when a Syllogiſm is ſaid to be in the firſt, ſecond, or third Figure; for, as there by the various application of the *Medium* or mean terme the *Figures* are conſtituted diverſe: ſo doth the difference of the *Datas* in a Triangle diſtinguiſh theſe *Triſſotetrall* Figures from one another, and (to continue yet further in the Syllogiſticall Analogy) are according to the ſeverall demands (when the *Datas* are the ſame) ſubdivided into *Moods*.

The firſt two vowels give notice of the *Data's*, and the third of what is demanded, ſo that *Uale* (and euphonetically pronounced *Vale*) which is the firſt Figure, ſhewes that the *Subtendent*, and one *Angle* are given, and that one of the containing *ſides* is required.

*Vemane* is the ſecond Figure, which pointeth out all thoſe problems wherein the *Hypotenuſa*, and one *Leg* are given, and an *Angle*, or the other *Leg* is required.

The third *Figure* is *Enave*, which comprehendeth all the Pro-

blems, wherein one of the *Ambients* is given with an Oblique *Angle*, and the *Subtendent*, or other *Ambient* required.

The fourth and laſt of the Rectangular Figures is *Ereva*, which ſtandeth for thoſe Datoquæres, wherein the including *Sides* are given and the *Subtendent* or an *Angle* demanded.

## Now let us come to the Moods of thoſe Figures.

THe firſt Figure *Vale* hath but one Mood, and therefore of as great extent as it ſelfe, which is *Upalem*; whoſe nature is to let us know, when a plane right angled Triangle is given us to reſolve, whoſe *Subtendent* and one of the *Obliques* is propoſed, and one of the *Ambients* required, that we muſt have recourſe unto its Reſolver, which being *Rad*—U—*Sapy* ☞ *Yr* ſheweth, that if we *joyne the artificiall Sine of the Angle oppoſite to the ſide demanded with the Logarithm of the Subtendent, the ſumme ſearched in the Canon of abſolute numbers will afford us the Logarithm of the ſide required.* The reaſon hereof is found in the ſecond Axiom, the firſt Conſonant of whoſe Directory evidenceth that *Upalem* is *Eproſos* Enodandum; for it is, *As the totall Sine, to the Hypotenuſa: ſo the Sine of the Angle oppoſite to the ſide required, is to the ſaid required ſide*, according to the nature of the foreſaid Axiom, whereupon it is grounded.

The ſecond Figure *Vemane* hath two Moods, *Ubeman* and *Uphener*; the firſt whereof comprehendeth all thoſe queſtions, wherein the *Subtendent* and an *Ambient* being given, an *Oblique* is required, and by its Reſolver *V*—*Rad*—*Eg* ☞ *So.* thus ſatisfieth our demand, that if we *ſubtract the Logarithm of the Subtendent from the ſumme of the Logarithms of the middle termes, we have the Logarithm of the Sine of the oppoſite Angle we ſeek for*; for it is, *As the Subtendent to the totall Sine, ſo the containing ſide given to the Sine of the oppoſite Angle required.* The reaſon likewiſe of this Analogy is found in the ſecond Axiom *Eproſo*, upon the which this *Mood* is grounded, as the ſecond Conſonant of its *Directory* giveth us to underſtand.

The ſecond Mood or Datoquære of this Figure is *Uphener*, which ſheweth that thoſe queſtions in plaine Triangles, wherein

wherein the *Hypotenuſa* and a *Leg* being given, the other *Leg* is demanded, are to be calculated by its *Reſolver*, which (becauſe the Canon of Logarithms cannot performe it at one operation, there being a neceſſity to find one of the oblique *Angles* before the fourth *terme* can be brought into an Analogie) alloweth two *Subſervients* for the atchievement thereof, *viz.*, *Vbeman*, the firſt Mood of the ſecond Figure, for the finding out of the *Angle*, and here (becauſe anterior in the work) called *Præſubſervient*: then *Vpalem*, the firſt Mood of all, for finding out of the *Leg* inquired, and here called *Poſſubſervient*, becauſe of its poſteriority in the operation: yet were it not for the facility which addition and ſubtraction only afford us in this manner of calculation, we might doe it with one work alone by the *Bucarnon* or Pythagoraſes *Diodot*, which plainly ſheweth us, that by *ſubducing the ſquare of the Leg given, from the ſquare of the Subtendent, we have for the remainder another ſquare, whoſe root is the ſide required.* The reaſon of this is in my Apodicticks: but that of the former Reſolver by two operations, is in the firſt Axiom, as by the firſt ſyllable of its *Directory* is manifeſt.

The third Figure is *Enave*, which hath two Moods, *Ekarul* and *Egalem*. The firſt comprehendeth all thoſe Problems, wherein one of the including ſides, and an Angle being given, the *Subtendent* is required, and by its Reſolver *Sapeg—Eg—Rad ☞ Vr*, ſheweth, that *if we ſubtract the Sine of the Angle oppoſite to the given ſide from the ſumme of the middle termes* (I meane the Logarithms of the one and the other) which are the totall Sine, and the Leg propoſed, *we ſhall have the Hypotenuſa required*; for it is, *As the Sine of the Angle oppoſite to the ſide given, to the foreſaid given ſide: ſo the totall Sine, to the Subtendent required.* The reaſon of this proportion is grounded on the ſecond Axiom *Epreſo*; for *K.* the third Conſonant of its *Directory*, giveth us to underſtand, that it is one of the Enodandas thereof.

The ſecond Mood of *Enave* is *Egalem*, which comprehendeth all thoſe *Problems*, wherein one of the *Ambients*, and an oblique *Angle* being given, the other *Ambient* is required: and by its Reſolver *Rad—Taxeg—Eg ☞ Yr* ſheweth, that if we *adde the Logarithm of the ſide given to the Logarithm of the Tangent of the Angle conterminall with that ſide, and from the ſumme if we cut off the firſt digit on the left hand* (which is equivalent to the ſubtracting

ing of the Radius whether double or ſingle) *The remainder will afford us a Logarithm* (ſo neare as the irrationality of the termes will admit) *in the Table of equall parts, expreſſive of the ſide required*; for it is, *As the whole Sine, to the Tangent of an Angle inſident on the given ſide: ſo the ſide propoſed, to the ſide required:* The reaſon hereof is grounded on the ſecond *Axiom*, for the fourth Conſonant of its *Directory* ſheweth, that *Egalem* is *Eproſos* enodandum.

The fourth Figure is *Ereva*, whoſe Moods are *Echemun* and *Etenar*.

The firſt, *viz. Echemun*, comprehendeth all thoſe Problems, wherein the two *Ambients* being given, the *Subtendent* is required, and (not being Logarithmically reſolvable in leſſe then two operations) hath for its *Præ* and *Poſſubſervients* the Moods of *Etenar* and *Ekarul*; for an Oblique *Angle* by this Method is to be ſearched before the *Subtendent* can be found out, and by reaſon of theſe ſeverall works, this *Mood* is grounded on the two firſt *Axioms*, and is an *Enodandum* partially depending on *Eradetul*, and *Eproſo*. Yet, if you will be pleaſed to be at the paines of extracting the *Square root*, you may have the *Subtendent* at one work by a Quadrobiquadræquation as the *Bucarnon* doth inſtruct us, whoſe demonſtration you have plainly ſet downe in the fourteenth of my *Apodicticks*.

The ſecond Mood of this Figure is *Etenar*, which includeth all thoſe queſtions wherein the two *containing Sides* being given, one of the *Obliques* is required, and by its *Reſolver E—Ge—Rad Toge* manifeſteth, that, *if from the Summe of the Radius and Logarithm of the ſide given, we ſubtract the Logarithm of the other propoſed ſide, the remainder will afford the Tangent of the Angle oppoſite to one of the given ſides*, the Complement of which Angle to a right one is alwayes the meaſure of the other Angle, by the fifth of my *Apodicticks*; for it is, *As the one Ambient is to the other Ambient, ſo the totall Sine to the Tangent of an Angle*; which found out, is either the Angle required, or the Complement thereof to a right Angle. The reaſon of this Analogie is grounded on the ſecond *Branch* of the firſt *Axiom*, as by the *Characteriſtick* of the *Directory* is perceivable enough to any induſtrious Reader.

Of

# Of the Planobliquangular Triangles there be foure Figures:

*Alaheme, Emenarole, Enerolome,* and *Erelea,*

THe firſt and laſt of theſe foure are *Monotropall* Figures, and have but each one Mood: but the other two have a couple a piece, ſo that for the *Planobliquangulars*, all the foure together afford us ſix *Datoquæres.*

The Mood of *Alaheme* is *Danarele*, which comprehendeth all thoſe *Problems*, wherein two *Angles* being given, and a *Side*, another *Side* is demanded, and by its Reſolver *Sapeg—Eg—Sapyr* ☞ *Yr*, ſheweth, that, if *to the ſumme of the Logarithm of the ſide given, and of the Sine of the Angle oppoſite to the ſide required, we adde the difference of the Secant complement from the Radius*, (by ſome called the Arithmeticall complement of the Sine, and in Maſter *Speidels* Logarithmicall Canon of *Sines, Tangents,* and *Secants* with good reaſon termed the *Secant*; for, though it doe not cut any Arch, thereby more Etymologically to deſerve the name of *Secant*, yet worketh it the ſame effect that the prolonged *Radius* doth) the operation will proceed ſo neatly, that if from theſe three Logarithms thus ſummed up, we onely cut off a *Digit* at the left hand, we will find as much by addition alone performed in this caſe, as if from the propoſed ſumme the Sine of the Angle had beene abſtracted; for the totall Sine thus *unradiuſed* is the Logarithm of the ſide required. But ſuch as are not acquainted with this compendious manner of calculating, or peradventure are not accommodated with a convenient Canon for the purpoſe, may, in Gods name, uſe their owne way, the Reſolver being of ſuch amplitude, that it extends it ſelfe to all ſorts of operations, whereby the truth of the fourth *Ternary* in this *Mood* may be attaind unto; for it is Analogiſed thus, *As the Sine of the Angle oppoſite to the ſide given is to the ſame given ſide: ſo the Sine of the Angle oppoſite to the ſide required, to the required ſide.* The reaſon of this proportion is grounded on the ſecond *Axiom*, the firſt *determinater* of whoſe *Directory* ſheweth, that *Danarele* is one of *Eproſos* Enodandas.

The ſecond Figure of the *Planobliquangulars* is *Emenarole*, whoſe Moods are *Therelabmo* and *Zelemabne.* The

The firſt comprehendeth all thoſe *Planobliquangular* Problems wherein two *ſides* being given with an *interjacent Angle*, an *oppoſite Angle* is demanded, and by its Reſolver *Aggres—Zes—Talfagros* ☞ *Talzos*, ſheweth, *that if from the ſumme of the Logarithm of the difference of the ſides, and Tangent of halfe the ſumme of the oppoſite Angles, be ſubduced the aggregat or ſumme of the Logarithms of the two propoſed ſides, the remainder thereof will preve the Logarithm of the Tangent of halfe the difference of the oppoſite Angles*; the which joyned to the one, and abſtracted from the other, affords us the meaſure of the Angle we require; for the *Theoreme* is, *As the aggregat of the given ſides, to the difference of theſe ſides: So the Tangent of halfe the ſumme of the oppoſite Angles, to the Tangent of halfe the difference of thoſe Angles*; which, without any more adoe, by ſimple Addition and Subtraction affordeth the *Angle* we demand. The third *Axiom* and the *Theorem* of the Reſolver of this Mood being but one and the ſame thing, I muſt make bold to remit you to my *Apodicticks* for the reaſon of the Analogie thereof, the onely determinater of whoſe Directory being θ. pointeth out the Mood of *Therelabmo* for the ſole *enodandum* appropriated thereunto.

The ſecond *Mood* of this Figure is *Zelemabne*, which involveth all the *Planobliquangulary* Problemes, wherein two *ſides* being given with the *Angle between*, the third ſide is demanded: and not being calculable by the *Logarithmicall Canon* in leſſe then two operations, becauſe it requireth the finding out of another *Angle* before it can fix upon the *ſide*, *Therelabmo* is allowed it for a *Præſubſervient*, by vertue whereof an oppoſite *Angle* is obtained, and *Danarele* for its *Poſſubſervient* and finall Reſolver, by whoſe meanes we get the ſide required. The reaſon of the firſt operation is grounded on the third *Axiom*, and of the ſecond operation on the ſecond: but becauſe this Mood is meerly a partiall *Enodandum*, neither of the foreſaid *Axioms* affordeth any *Directory* concerning it, otherwiſe then in the two *Subſervients* thereof.

The third Figure is *Enerolome*, whoſe two Moods are *Xemenaro* and *Shenerolem*.

The firſt *Mood* of this *Figure* includeth all thoſe *Planobliquangularie* Problems, wherein two *ſides* being given, with an *oppoſite Angle*, another *oppoſite Angle* is demanded, and by its Reſolver

*E—Sog—Ge☞* So, ſheweth, that *if from the aggregat of the Logarithm of one of the given ſides, and that of the Sine of the oppoſite Angle propoſed, we ſubtract the Logarithm of the other given ſide, the reſidue will afford us the Logarithm of the Sine of the oppoſite Angle required*; for it is Analogiſed thus, *As one of the ſides, to the Sine of the oppoſite Angle given: ſo the other ſide propoſed, to the Sine of the oppoſite Angle required.* The reaſon of this proportion is from the ſecond *Axiom*, the ſixth characteriſtick of whoſe *Directory* importeth, that *Xemenoro* is one of *Eproſos* enodandas.

The ſecond Mood of this Figure is *Shenerolem*, which containeth all thoſe Planobliquangularie Problems, wherein two *ſides* being given with an *oppoſite Angle*, the third ſide is demanded, which not being findable by the Logarithmicall Table upon the foreſaid *Datas* in leſſe then two operations (becauſe an *Angle* muſt be obtained firſt before the *ſide* can be had) *Xemenoro* Præſubſerves it for an *Angle*, and *Danarele* becomes its *Poſſubſervient* for the *ſide* required. The reaſon of both theſe operations is founded on the ſecond *Axiom*, the laſt *Characteriſtick* of whoſe *Directory* inrolleth *Shenerolem* for one of *Eproſos* enodandas.

The fourth figure is *Erelea*, which, being *Monotropall*, hath no Mood but *Pſerelema*.

This *Pſerelema* encompaſſeth all thoſe *Planobliquangulary* Problems wherein the three *ſides* being propoſed, an *Angle* is required. This *Datoquare* not being reſolvable by the *Logarithms* in leſſe then two operations, becauſe the Segments of the *Baſe*, or ſuſtaining ſide muſt needs be found out, that by demitting of a *Perpendicular* from the *top Angle*, we may hit upon the *Angle* demanded: the Reſolver for the Segments is *Ba—Gres—Zes☞ Zisu*, whereby we learne, that *if from the Logarithm of the ſumme of the ſides, joyned to the Logarithm of the difference of the ſides, we ſubtract the Logarithm of the Baſe, the remainder is the Logarithm of the difference of the Segments*, which *difference* being taken from the whole *Baſe*, halfe the *difference* proves to be the leſſer *Segment*. This *Theorem* being thus the Præſubſervient of this *Mood*, its Poſſubſervient is *Vbeman*, whoſe generall Reſolver *V—Rad—Eg☞ Sor*, is particulariſed for this caſe *Uxing—Rad—Iug☞ Sor*, which ſheweth, that *if from the ſumme of the Logarithms of the totall Sine, and of one of the Segments given, we ſubduce the Loga-*

*rithm of the Hypotenuſa conterminall with the Segment propoſed, the remainer will be the Logarithm of the Sine of the oppoſite Angle required*; for the demitting of the Perpendicular opens a way to have the *Theorem* to be firſt in generall propounded thus, *As the Subtendent to the totall Sine, ſo the containing ſide given to the Sine of the Angle required*: or in particular thus, *As the Sine of the Coſubtendent adjoyning the Segment given is to the Radius, ſo is the ſaid Segment propoſed to the Sine of the Angle required.*

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# Thus farre for the calculating of plaine Triangles, both right and oblique: now follow the Sphericals.

THere be three principall *Axioms* upon which dependeth the reſolving of *Spherical* Triangles, to wit, *Suproſca*, *Sbaprotca*, and *Seproſo*.

The firſt *Maxime* or *Axiom*, *Suproſca*, ſheweth, that of ſeverall *rectangled* Sphericals, which have one and the ſame *acute Angle* at the *Baſe*, the *Sines of the Hypotenuſas are proportionall to the Sines of their Perpendiculars*; for, from the ſame inclination every where of the one plaine to the other, there ariſeth an equiangularity in the two rectangles, out of which we may confidently inferre the homologall ſides (which are the Sines of the Subtendents, and of the Perpendiculars of the one, and the other) to be amongſt themſelves proportionall. Its Directory is *Uphugen*, by the which we learn, that *Uphanep*, *Ugemon*, and *Enarul*, are its three enodandas.

The ſecond *Axiom* is *Sbaprotca*, whereby we learne, that in all rectangled Sphericals that have one and the ſame acute Angle at the Baſe, *the Sines of the Baſes are proportionall to the Tangents of their Perpendiculars*: which Analogie proceedeth from the equiangularity of ſuch rectangled Sphericals, by the ſemblable inclining of the plaine towards them both. This proportion nevertheleſſe will never hold betwixt the Sines of the Baſes, and the Sines of their Perpendiculars; becauſe, if the Sines of the Baſes were proportionall to the Sines of the Perpendiculars (the Sines

of

of the Perpendiculars being already demonſtrated propotionall to the Sines of the Subtendents) either the *Sine* of the Perpendicular, or the *Sine* of the Baſe would be the cord of the ſame Arch, whereof it is a Sine; which is impoſſible, by reaſon that nothing can be both a whole, and a part, in regard of one and the ſame thing; and therefore doe we onely ſay, that *the Sines of the Baſes, and Tangents of the Perpendiculars, and contrarily, are proportionall.* Its Directory is *Pubkutethepſaler*, which ſheweth, that *Upalam, Ubamen, Vkelamb, Etalum, Ethaner, epſoxer, Alamus*, and *Erelam*, are the eight Enodandas thereupon depending.

The third *Axiom* is, that *the Sines of the ſides are proportionall to the Sines of their oppoſite Angles:* the truth whereof holds in all Spherical Triangles whatſoever; which is proved, partly out of the proportion betwixt the Sines of the Perpendiculars ſubſterned under equall Angles, and the Sines of the Hypotenuſas: and partly, by the Analogy, that is betwixt the Sines of the Angles ſuſtained by ſeverall Perpendiculars, demitted from one point, and the Sines of the Perpendiculars themſelves. The Directory of this *Axiom* is *Vchedezexam*, whereby we know that *Uchener, Edamon, Ezolum, Exoman*, and *Amaner*, are the five Enodandas thereof.

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# The Orthogonoſpherical Table conſiſteth of theſe ſix Figures:

*Valamenep, Vemanore, Enarulome, Erolumane, Achave*, and *Eſheva.*

THe firſt Figure, *Valamenep*, comprehendeth all thoſe queſtions, wherein the *Subtendent, and an Angle being given, either another Angle, or one of the Ambients is demanded.*

Of this Figure there be three Moods, *viz.* *Upalam, Ubamen*, and *Uphanep.* The firſt, to wit *Upalam*, containeth all thoſe Orthogonoſpherical Problems, wherein the *Subtendent and one oblique Angle being given, another oblique Angle is required*, and by its Reſolver *Torb—Tag—Nu ☞ Mir*, ſheweth, that the ſumme of the Sine complement of the Subtendent ſide and Tangent of the Angle given, (the Logarithms of theſe are alwayes to be underſtood)

a digit being preſcinded from the left, is equall to the Tangent complement of the Angle required; for the propoſition goeth thus, *As the Radius, to the Tangent of the Angle given: ſo the Sine complement of the Subtendent ſide, to the Tangent complement of the Angle required:* and becauſe Tangents, and Tangent complements are reciprocally proportionall, inſtead of *To—Tag—Nu☞Mir*, or, *To—Lu—Mag☞Tir*, which (for that the Radius is a meane proportionall betwixt the *L.* and *N*; the *T.* and *M*) is all one for inferring of the ſame fourth proportionall, or foreſaid quæſitum) we may ſay, *Mag—Nu—To☞Mir*, that is, *As the Tangent complement of the given Angle to the Coſine of the Subtendent, ſo the totall Sine to the Antitangent of the Angle demanded*; for the totall Sine being, as I have told you, a meane proportionall betwixt the Tangents and Cotangents, the ſubtracting of the Cotangent, or Tangent complement from the ſumme of the Radius, and Antiſine reſiduats a Logarithm equall to that of the remainder, by abſtracting the Radius from the ſum of the Coſine of the ſubtendent, and Tangent of the Angle given, either of which will fall out to be the Antitangent of the required Angle.

*Notandum.*

[Here alwayes is to be obſerved, that the ſubtracting of Logarithms may be avoyded, by ſubſtituting the *Arithmeticall complement thereof*, to be added to the Logarithms of the two middle proportionals (which Arithmeticall complement (according to *Gellibrand*) is nothing elſe, but the difference betweene the Logarithm to be ſubtracted, and another conſiſting of an unit, or binarie with the addition of cyphers, that is the ſingle, or double Radius) for ſo the ſum of the three Logarithms, cutting off an unit, or binarie towards the left hand, will ſtill be the Logarithm of the fourth proportionall required.

For the greater eaſe therefore in Trigonometricall computations, ſuch a Logarithmicall Canon is to be wiſhed for, wherein the Radius is left out of all the Secants, and all the Tangents of Major Arches, according to the method preſcribed by Mr. *Speidel*, who is willing to take the paines to make ſuch a new Canon, better then any that ever hitherto hath beene made uſe of, ſo that the publike, whom it moſt concerneth, or ſome potent man, well minded towards the Mathematicks, would be ſo generous,

as

as to releeve him of the charge it muſt needs coſt him; which, conſidering his great affection to, and ability in thoſe ſciences, will certainly be as ſmall a ſumme, as poſſibly he can bring it to.]

*This Parentheſis, though ſomewhat with the longeſt, will not (I hope) be diſpleaſing to the ſtudious Reader.*

The ſecond Mood of the firſt Figure is *Ubamen*, which comprehendeth all thoſe Problems, wherein the *Subtendent, and one oblique Angle being given, the Ambient adjoyning the Angle given is required*, and by its Reſolver, *Nag—Mu—Torp ☞ Myr*, ſheweth, that, *if to the ſumme of the Logarithms of the two middle proportionals, we adde the Arithmeticall complement of the firſt, the cutting off the Index from the Aggregat of the three, will reſiduat the Tangent complement of the ſide required*: and therefore with the totall Sine in the firſt place, it may be thus propounded, *Torp—Mu—Lag ☞ Myr*; for the firſt Theorem being, *As the Sine complement of the Angle given, to the Tangent complement of the ſubtendent ſide: ſo the totall Sine, to the Tangent complement of the ſide required*: juſt ſo the ſecond Theorem, which is that refined, is, *As the totall Sine, to the Tangent complement of the Subtendent: ſo the Secant of the given Angle, to the Tangent complement of the demanded ſide.* Here you muſt conſider, as I have told you already, that of the whole Secant I take but its exceſſe above the Radius, as I doe of all Tangents above 45. Degrees; becauſe the cutting off the firſt digit on the left, ſupplieth the ſubtraction, requiſite for the finding out of the fourth proportionall; ſo that by addition onely the whole operation may be performed, of all wayes the moſt ſuccinct and ready. Otherwiſe, becauſe of the totall Sines meane proportionality betwixt the Sine complement, and the Secant; and betwixt the Tangent, and Tangent complement, it may be regulated thus, *To—Tu—Nag ☞ Tyr*, that is, *As the Radius, to the Tangent of the Subtendent, ſo the Sine complement of the Angle given, to the Tangent of the ſide required.* The reaſon of the reſolution both of this, and of the former Datoquære, is grounded on the ſecond Axiom, and the proportion that, in ſeverall rectangled Sphericals which have the ſame acute Angle at the Baſe, is found betwixt the Sines of their Perpendiculars, and Tangents of their Baſes, as is ſhewne you by the two firſt Conſonants of the Directory of *Sbaprotca*.

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The third and laſt Mood of the firſt Figure is *Ukphaner*, which comprehendeth all thoſe Problems, wherein *the Hypotenuſa, and one of the obliques being given, the oppoſite Ambient is required*, and by its Reſolver *Tol—Sag—Su☞Syr*, ſheweth, that, *if we adde the Logarithms of the Sine of the Angle, and Sine of the Subtendent, cutting off the left Supernumerarie digit from the ſumme, it gives us the Logarithm of the Sine of the ſide demanded*; for it is, *As the totall Sine, to the Sine of the Angle given: ſo the Sine of the ſubtendent ſide, to the Sine of the ſide required*: and becauſe by the *Axiom* of *Ruterſt*, it was proved, that *when the Sine of any Arch is made Radius, what was then the totall Sine, becomes a Secant* (and therefore Secant complement of that Arch) inſtead of *Tol—Sag—Su☞Syr*, we may ſay, *To—Ru—Rag☞Ryr*, that is, *As the totall Sine, is to the Secant complement of the ſubtendent: ſo the Secant complement of the Angle given, to the Secant complement of the ſide demanded.* The reſolution of this Datoquære by Sines, is grounded on the firſt Axiom of Sphericals, which elucidats the proportion betwixt the Sines of the Hypotenuſas, and Perpendiculars, as it is declared to us by the firſt ſyllable of *Suproſcas* Directory.

The ſecond Figure is *Uemanore*, which containeth all thoſe Orthogonoſphericall queſtions, wherein *the ſubtendent, and an Ambient being propoſed, either of the obliques, or the other Ambient is required*, and hath three Moods, *viz. Ukelamb*, *Ugemon*, and *Uchener*.

The firſt Mood *Ukelamb* comprehendeth all thoſe Orthogonoſphericall Problems, wherein the *ſubtendent, and one including ſide being given, the interjacent Angle is demanded*, and by its Reſolver *Meg—Torp—Mu☞Nir* (or becauſe of the totall Sines mean proportion betwixt the Tangent, and Tangent complement) *Torp-Teg—Mu☞Nir* (which is the ſame in effect) ſheweth, that if *from the ſumme of the Logarithms of the middle termes*, (which in the firſt Analogy is the Radius, and Tangent complement of the ſubtendent) *we ſubtract the Tangent complement of the given Ambient*: or, in the ſecond order of proportionals, *joyne the Tangent of the ſide given, to the Tangent complement of the ſubtendent, and from the ſum cut off the Index* (if need be) both will tend to the ſame end, and *produce for the fourth proportionall, the Sine complement of the Angle required*; for to ſubtract a Tangent complement from the Radius, and another

ther number joyned together, whether that Tangent complement be more or leſſe then the Radius, it is all one, as if you ſhould ſubtract the Radius from the ſaid Tangent complement, and that other number; becauſe the Tangent (or rather Logarithm of the Tangent; for ſo it muſt be alwayes underſtood, and not onely in Tangents, but in Sines, Secants, Sides, and Angles, though for brevity ſake the word Logarithm be oftentimes omitted) becauſe I ſay, the Logarithms of the Tangent, and Tangent complement together, being the double of the Radius; if firſt the Tangent complement ſurpaſſe the Radius, and be to be ſubtracted from it, and another number, it is all one, as if from the ſaid number you would abſtract the Radius, and the Tangent complements exceſſe above it, ſo that the Radius being in both, there will remaine a Tangent with the other number. Likewiſe, if a Tangent complement, leſſe then the Radius, be to be ſubtracted from the ſumme of the Radius, and another Logarithm; it is yet all one, as if you had ſubtracted the Radius from the ſame ſumme; becauſe, though that Tangent complement be leſſe then the Radius: yet, that parcell of the Radius which was abſtracted more then enough, is recompenſed in the Logarithm of the Tangent to be joyned with the other number; for, from which ſoever of the Tangents the Radius be ſubduced, its Antitangent is remainder: both which caſes may be thus illuſtrated in numbers; and firſt, where the Tangent complement is greater then the Radius, as in theſe numbers 6. 4. 3. 1. and 4. 2. 3. 1. where, let 6. be the Tangent complement, 4. the Radius, 3. the number to be joyned with the Radius, or either of the Tangents, and 1. the remainer; for 4. and 3. making 7. if you abſtract 6. there will remaine 1. Likewiſe 2. and 3. making 5. if you ſubtract 4. there will remaine 1. Next, if the Tangent complement be leſſe then the Radius, as in 2. 4. 3. 5. and 4. 6. 3. and 5. where, let 2. be the Tangent complement; for if from 4. and 3. joyned together, you abſtract 2. there will remaine 5. which will alſo be the remainder, when you ſubtract 4. from 6. and 3. added together. Now to make the ſame Reſolver (the variety whereof I have beene ſo large in explaining) to runne altogether upon Tangents, inſtead of *Meg—To—Mu. ✠ Nir*, that is, *As the Tangent complement of the ſide given, is to the totall Sine: ſo the Tangent*

*gent complement of the ſubtendent ſide, to the Sine complement of the Angle required*, we may ſay, *Tu—Teg—To☞Nir*; that is, *As the Tangent of the ſubtendent, is to the Tangent of the given ſide; ſo the totall Sine, to the Sine complement of the Angle required.* All this is grounded on the ſecond *Axiom Sbaprotca*, and upon the reciprocall proportion of the Tangents and antitangents, as is evident by the third characteriſtick of its Directory.

The ſecond Mood of *Vemanore* is *Ugemon*, which comprehendeth all thoſe orthogonoſphericall problems, wherein the *ſubtendent with an Ambient being given, an oppoſite oblique is required*, and by its Reſolver, *Su-Seg-Tom☞Sir*, or (by putting the Radius in the firſt place, according to *Uradeſſo*, the firſt branch of the firſt axiom of the Planorectangulars) *To-Seg-Ru☞Sir*, ſheweth, that the *ſumme of the ſide given, and ſecant of the ſubtendent* (the Supernumerarie digit being cut off) *is the ſine of the Angle required*; for the Theorem is, *As the ſine of the ſubtendent, to the ſine of the ſide given; ſo the Radius, to the ſine of the Angle required:* or, *As the totall ſine, to the ſine of the ſide given: ſo the ſecant complement of the ſubtendent, to the ſine of the angle required:* or, changing the ſines into ſecant complements, and the ſecant complements into ſines, we may ſay, *To--Su—Reg☞Rir*; becauſe, betwixt the ſine and ſecant complement, the Radius is a middle proportion. Other varieties of calculation in this, as well as other problems, may be uſed; for, beſides that every proportion of the Radius to the ſine, Tangent, or ſecant, and contrarily, may be varied three manner of wayes, by the firſt Axiom of Plaine triangles, the alteration of the middle termes may breed ſome diverſity, by a permutat, or perturbed proportion, which I thought good to admoniſh the Reader of here, once for all, becauſe there is no problem, whether in Plaine, or Spherical triangles, wherein the Analogie admitteth not of ſo much change. The reaſons of this Mood of *Ugemon*, depend on the Axiom of *Suproſca*, as the ſecond characteriſtick of *Uphugen* ſeemeth to inſinuate.

The laſt Mood of the ſecond figure is *Uchener*, which comprehendeth all thoſe problems, wherein *the ſubtendent, & one Ambient being given, the other Ambient is Required*, and by its Reſolver, *Neg—To—Nu☞Nyr*, or, *To--le-Nu☞Nyr*, ſheweth, that *the ſumme of the ſine complement of the ſubtendent, and the ſecant of the given*

*given ſide* (which is the Arithmeticall complement of its Antiſine) *giveth us the ſine complement of the ſide deſired*, the Index being removed; for the theorem is, As *the ſine complement of the given ſide, to the total ſine: ſo the ſine complement of the ſubtendent, to the ſine complement of the ſide required*: or more refinedly, As *the Radius, to the ſine complement of the ſubtendent: ſo the ſecant of the Leg given, to the ſine complement of the ſide required*: and beſides other varieties of Analogie, according to the Axiom of *Rulerſt*, by making uſe of the reciprocall proportion of the ſine-complements with the ſecants we may ſay, *To-Ne Lu-☞☜ Lyr*, that is, As *the totall ſine, is to the ſine complement of the given ſide: ſo the ſecant of the ſubtendent, to the ſecant of the ſide required*. The reaſon of this Datoqueres Reſolution is in *Seproſo* the third Axiom of the Sphericals, as is manifeſt by the firſt figurative of its Directorie *Uchedezexam*.

The third figure is *Enarrulome*, whoſe three Moods are *Etalum*, *Edamon*, and *Ethaner*.

This *figure* comprehendeth all thoſe orthogonoſphericall queſtions, wherein *one of the Ambients with an Adjacent angle is given, and the ſubtendent, an oppoſite angle, or the other containing ſide is required*.

Its firſt Mood *Etalum*, involveth all thoſe Orthogonoſphericall problems, wherein *a containing ſide, with an inſident angle thereon is propoſed, and the hypotenuſa demanded*: and by its reſolver *Torpme--nag☞☜ mur* or (by inverting the demand upon the Scheme) *Tolp---me--nag☞☜ mur* ſheweth, that the cutting of the firſt left digit, from the ſumme of the Tangent complement of the Ambient propoſed, and the ſine complement of the given angle, affords us the Tangent complement of the ſubtendent required; for the theorem goes thus, As *the totall ſine, to the tangent complement of the given ſide: ſo the ſine complement of the angle given, to the tangent complement of the hypotenuſa required*. And becauſe the totall ſine, hath the ſame proportion to the tangent complement, which the ſine, hath to the ſine complement, we may as well ſay, *To-meg-Sa☞☜ nur*, that is, As the *Radius to the tangent complement of the Ambient ſide: ſo the ſine of the angle given, to the ſine complement of the ſubtendent required*. The progreſſe of this Mood, dependeth on the Axiom of *Sbaprotca*, as you may perceive by the fourth conſonant of its directorie *Pubkutethepſaler*.

The ſecond *Mood* of the third *Figure*, is *Edamon*, which comprehendeth all thoſe Orthogonoſpherical Problems, *wherein an Ambient and an Adjacent angle being given, the oppoſite oblique* (viz. *the angle under which the Ambient is ſubtended*) *is required*, and by its Reſolver *To-Neg-Sa ⋋ Nir*, ſheweth that the Addition of the Coſine of the Ambient, and of the ſine of the Angle propoſed, affordeth us (if we omit not the uſuall preſection) the Coſine of the Angle we ſeek for; for it is, *As the Radius to the Coſine, or ſine complement of the given ſide: ſo the ſine of the Angle propoſed to the Antiſine or ſine complement of the Angle demanded*: now the Radius being alwayes a meane proportionall betwixt the Sine complement, and the Secant, we may for *To—Neg—Sa ⋋ Nir*, ſay, *To—Leg—Ra ⋋ Lir*, or *To—Rag—Le ⋋ Lir*: that is, *As the totall Sine to the Secant, or cutter of the ſide given, or to the Coſecant, or Secant complement of the given Angle; ſo is the Secant complement of the Angle, or Secant of the ſide, to the Secant, or cutter of the Angle required.* The reaſon of all this is grounded on *Seproſo*, becauſe it runneth upon the proportion betwixt the Sines of the ſides, and the Sines of their oppoſite Angles, as is perſpicuous to any by the ſecond ſyllable of the Directory of that *Axiome*.

The laſt *Mood* of the third *Figure* is *Ethaner*, which comprehendeth all thoſe Orthogonoſpherical Problems, wherein an *Ambient with an Oblique* annexed thereto, *is given, and the other Arch about the right Angle is required*, and by its Reſolver, *Torb—Tag—Se ⋋ Tyr*, ſheweth, that if we joyne the Logarithms of the two middle proportionals, which are the Tangent of the given Angle, and the Sine of the ſide, the uſuall preſection being obſerved, we ſhall thereby have the Tangent, or toucher of the *Ambient* ſide deſired; for it is, *As the Radius to the Tangent of the Angle given, ſo the Sine of the containing ſide propoſed to the ſide required*: And becauſe the Tangent complement, and Tangent are reciprocally proportionall, the Sine likewiſe, and Secant complement, for *To—Tag—Se ⋋ Tyr*, we may ſay, keeping the ſame proportion, *To—Reg—Ma ⋋ Myr* that is, *As the Radius, to the Secant complement of the given ſide: ſo the Tangent complement of the Angle propoſed, to the Tangent complement of the ſide required.* The truth of all theſe operations dependeth on *Shaprotca*, the

ſecond

ſecond *Axiome* of the Spherical, as is evidenced by 9. the fifth characteriſtick of its Directory *Pubkuretheſaler.*

The fourth *Figure* is *Erollumane*, which includeth all Orthogonoſpherical queſtions, wherein *an Ambient, and an oppoſite oblique being given, the ſubtendent, the other oblique, or the other Ambient is demanded:* It hath likewiſe, conforme to the three former *Figures*, three *Moods* belonging to it; the firſt whereof is *Ezolum.*

This *Ezolum* comprehendeth all thoſe Orthogonoſpherical, Problems, wherein *one of the Legs, with an oppoſite Angle being given, the Subtendent is required*, and by its Reſolver, *Sag—Sep—Rad☞Snr*, or by putting the Radius in the firſt place, *To—Se—Rag☞Snr*, ſheweth, that the abſtracting of the Radius from the ſum of the Sine of the ſide, and Secant complement of the Angle given, reſiduats the Sine of the hypotenuſa required; for it is, *As the Sine of the Angle given, to the Sine of the oppoſite ſide: ſo the Radius to the Sine of the ſubtendent:* or more refinedly, *As the totall Sine, to the Sine of the ſide: ſo the Secant complement of the Angle given, to the Sine of the ſubtendent ſide:* And becauſe of the Sines and Antiſecants, or Secant complements reciprocall proportionality, *To—Sag—Re☞Rn*, that is, *As the Radius, to the Sine of the Angle given: ſo the Secant complement of the propoſed ſide, to the Secant complement of the ſubtendent required.* The reaſon of all this is grounded on the third Axiom *Seproſo*, as is made manifeſt by the third Syllable of its Directory.

The ſecond *Mood* of this *Figure* is *Exoman*, which comprehendeth all thoſe Problems, wherein *a containing ſide, and an oppoſite oblique being given, the adjacent oblique is required:* and by its Reſolver, *Ne—To—Nag☞Sir*, or more refinedly, *To—Le—Nag☞Sir*, ſheweth, that the ſumme of the Sine of the Angle, together with the Arithmeticall complement of the Antiſine of the Leg, (which in the Table I have ſo much recommended unto the Reader, is ſet downe for a Secant) the uſuall preſection being obſerved, affordeth us the Sine of the Angle required, and becauſe of the reciprocall proportion betwixt the Sine complement, and Secant; and betwixt the Sine, and Secant complement, the Theorem may be compoſed thus: *To—Neg—La☞Rir*, that is, *As the Radius, to the Sine complement of the given ſide: ſo the Secant of the Angle propoſed, to the Secant complement of the*

*Angle demanded.* The reason of this is likewise grounded on *Seposo*, as you may perceive by the fourth characteristick of its Directory.

The last *Mood* of this Figure is *Epsoner*, which containeth all those Orthogonospherical Problems, wherein an *Ambient* and an opposite *Oblique* being given, the other *Ambient* is demanded, and by its Resolver, *Tag—Tolb—Te☞ Syr*, or more elaboured, *Tolb—Mag—Te☞ Syr*, sheweth, that the præscinding of the Radius from the summe of the Tangent of the side, and Antitangent of the given Angle, residuats the Sine of the side required; for it is, As *the Tangent of the Angle proposed, to the totall Sine: so the Tangent of the given side, to the Sine of the side demanded:* or, *As the Radius, to the Tangent complement of the Angle given: so the Tangent of the given side, to the Sine of the side required:* and because of the reciprocall Analogy betwixt the Tangents, and Co-tangents: and betwixt the Sines, and Co-secants, we may with the same confidence, as formerly, set it thus in the rule, *To—Meg—Ta☞ Ryr*, and it will find out the same quæsitum. The reason of the operations of this *Mood*, because of the ingrediencie of Tangents dependeth on *Shaprotca*, as is perceivable by the sixth determinater of its Directory *Pubkutethepsaler.*

The fifth *Figure* of the Orthogonospherical is *Achave*, which containeth all those Problems, wherein the *Angles being given, the subtendent or an Ambient is desired*, and hath two Moods *Alamun*, and *Amaner*.

*Alamun* comprehendeth all those Problems, wherein the *Angles being proposed, the Hypotenusa is required, and by its Resolver Tag—Torb—Ma☞ Nur*, or more compendiously, *Torb—Mag—Ma☞ Nur*, sheweth, that the summe of the Co-tangents, not exceeding the places of the Radius, is the Sine complement of the subtendent required; for it is, As *the Tangent of one of the Angles, to the Radius: so the Tangent complement of the other Angle, to the Sine complement of the Hypotenusa demanded:* or, As *the totall Sine, to the Tangent complement of one of the Angles: so the Tangent complement of the other Angle, to the Sine complement of the subtendent we seek for.* The running of this *Mood* upon Tangents, notifieth its dependance on *Shaprotca*, as is evident by the seventh determinater of the Directory thereof.

The

The second *Mood* of this Figure is *Amaner*, which comprehendeth all those Orthogonospherical Problems, wherein *the Angles being given, an Ambient is demanded*, and by its Resolver, *Say—Nag—Tω☞ Nyr*, or more perspicuously, *Tω—Noy—Ray☞ Nyr*, sheweth, that the summe of the Logarithms of the Antisine of the Angle opposite to the side required, and the Arithmeticall complement of the Sine of the Angle, adjoyning the said side, which we call its Secant complement, with the usuall presection, is equall to the Sine complement of the same side demanded; for it is, As *the Sine of the Angle adjoyning the side required, to the Antisine of the other Angle: so the totall sine, to the Antisine of the side demanded*: or, As *the Radius, to the Antisine of the Angle opposite to the demanded side: so the Antisecant of the Angle conterminat with that side, to the Antisine of the side required*: and because of the Analogy betwixt the Antisines, and Secants: and likewise betwixt the Antisecants, and Sines, we may expresse it, *To—Say—La☞ Lyr*; that is, As *the Radius, to the Sine of the Angle insident on the required side: so the Secant of the other given Angle, to the Secant of the side that is demanded.* Here the Angulary intermixture of proportions giveth us to understand, that this *Mood* dependeth on *Seproso*, as is manifested by the last characteristick of *Uchedezexam* the Directory of this *Axiom*.

The sixth and last Figure is *Escheva*, which comprehendeth all those Problems, wherein the two containing sides being given, either the *subtendent*, or an *Angle* is demanded: it hath two *Moods*, *Enerul* and *Erelam*.

The first *Mood* thereof *Enerul*, containeth all such Problems as having the *Ambients* given, require the *subtendent*, and by its Resolver, *Ton—Neg—Ne☞ Nur*, sheweth, that the *summe of the Logarithms of the Cosines of the two Legs unradiated, is the Logarithm of the Co-sine of the subtendent*; for it is, As *the totall Sine, to the Co-sine of one of the Ambients: so the Co-sine of the other including Leg given, to the Co-sine of the required subtendent*; and because of the Co-sinal, and Secantine proportion, we may safely say, *To—Leg—Le☞ Lur*. That is, As *the Radius to the Secant of one shanke or Leg: so the secant of the other shanke or Leg, to the secant of the Hypotenusa demanded.* The coursing thus upon Sines, and their proportionals evidenceth that this Mood dependeth on *Suprosca*, the

the firſt of the Sphericall Axioms, which is pointed at by the third and laſt characteriſtick of *Uphugen* the directorie thereof.

The ſecond *Mood* of the laſt *figure*, and conſequently the laſt *Mood* of al the *Orthogonoſphericals*, is *Erelam*, which comprehendeth all thoſe orthogonoſpherical problems, wherin the two containing *ſides* being propoſed, an *Angle* is demanded, and by its Reſolver, *Sei—Teg--Torb ☞ Tir*, or by primifying the Radius, *Torb--Tepi-Rexi ☞ Tir*, giveth us to underſtand, that the cutting off the *Radius* from the ſumme of the *Tangent* of the *ſide* oppoſite to the *Angle* demanded, and the *coſecant* of the *ſide* conterminat with the ſaid *Angle*, reſiduats the *touch-line* of the *Angle* in queſtion; for it is, *As the ſine of the ſide adjoyning the Angle required, to the tangent of the other given ſide: ſo the Radius to the tangent of the Angle demanded*: or, *As the totall ſine to the Tangent of the Ambient oppoſite to the angle ſought: ſo the Antiſecant of the Leg adjacent to the ſaid asked Angle, to the Tangent or toucher thereof*: and becauſe *Sines* have the ſame proportion to *coſecants*, which *Tangents* have to *Cotangents*, we may ſay, *To—Sei—me ☞ mir*, that is, *As the Radius to the ſine of the ſide conterminat with the angle required: ſo the Cotangent of the other Leg, to the Cotangent of the Angle ſearched after*: or yet more profoundly by an Alternat proportion, changing the relation of the fourth proportionall, although the ſame formerly required Angle, thus, *To—Rei—me ☞ mor*, that is, *As the Radius to the Antiſecant of the ſide adjacent to the Angle ſought for, ſo the Antitangent of the other ſide, to the Antitangent of that ſides oppoſit Angle*, which is the Angle demanded. The reaſon hereof is grounded on *Sbaprotca*; for the Tangentine proportion of the terms of this *Mood* ſpecifieth its dependance on the ſecond Axiom, which is ſhowen unto us by the eight and laſt characteriſtick of its directorie *Pubkutethepſaler*.

Here endeth the doctrine of the right-Angled ſphericalls, the whole diatypoſis wherof is in the *Equiſolea* or hippocrepidian diagram, whoſe moſt intricate amfractuoſities, renvoys, various mixture of analogies, and perturbat ſituation of proportionall termes, cannot chooſe but be pervious to the underſtanding of any judicious Reader that hath peruſed this Comment aright. And therefore let him give me leave (if he think fit) for his memorie ſake, to remit him to it, before he proceed any further.

The

# The Loxogonoſpherical Triangles, whether Amblygonoſpherical or Oxygonoſpherical, are either Monurgetick or Diſergetick.

THe *Monurgetick* have two figures, *Datamiſta* and *Datapura*.

*Datamiſta* is of all thoſe Loxogonoſpherical Monurgetick problems, wherein the *Angles* and *ſides* being intermixedly given, (and therefore one of them being alwaies of another kind from the other two) either an *Angle*, or a *ſide* is demanded: it hath two Moods, *Lamaneprep*, and *Menerolo*.

The firſt Mood *Lamaneprep*, comprehendeth all thoſe Loxogonoſpherical problems, wherein two *angles* being given, and an *oppoſit ſide*, another *oppoſit ſide* is demanded, and by its Reſolver, *Sapeg—Se—Sapy—☞Syr*, ſheweth, that if to the Logarithms of the *ſine* of the *ſide* given, and *ſine* of the *Angle* oppoſit, to the *ſide* required, we joyne the Arithmeticall complement of the *ſine* of the *Angle* oppoſit, to the propoſed *ſide* (which is the refined *Antiſecant*) we will thereby attain to the knowledge of the *ſine* of the *ſide* demanded. The reaſon of this is grounded on the third Axiom, *Seproſo*, as you may perceive by the firſt ſyllable of the Obliquangularie directory, *Lame*.

The ſecond *Mood* of this *figure* is *Menerolo*, wich comprehendeth all thoſe Amblygonoſpherical problems, wherein two *ſides* being given with an oppoſit *angle*, another oppoſit *angle* is demanded, and by its Reſolver *Sepag—Sa—Sepi☞Sir*, ſheweth, that if to the ſumme of the Logarithms of the *ſine* of the given *angle*, and *ſine* of the *ſide* oppoſit to the *angle* required, we joyne the Arithmeticall complement of the *ſine* of the *ſide* oppoſit to the given *angle* (which is the refined *Coſecant* of the ſaid *angle*) it will afford us the *ſine* of the *angle* required. The reaſon of this operation is grounded on the third Axiom of Sphericalls, a progreſſe in *ſines* ſhewing clearely, how that both this, and the former, doe totally depend on the Axiom of *Seproſa*, as is evident by the ſecond ſyllable of its directorie, *Lame*.

The ſecond *figure* of the Monurgetick Loxogonoſpherical treateth

treateth of all thoſe queſtions, wherein the *Datas* being either *ſides* alone, or *Angles* alone, an *Angle* or a *ſide* is demanded. This *Figure* of *Datapura* is divided into two *Moods*, viz. *Nerelema*, and *Kalamane*, which are of ſuch affinity, that upon one and the ſame *Theorem* dependeth the *Analogy* that reſolveth both.

The firſt *Mood* thereof, *Nerelema*, comprehendeth all thoſe Problems, wherein the three *ſides* being given, an *Angle* is demanded, and is the third of the Monurgeticks, as by its *Characteriſtick* the third Liquid is perceivable.

The curteous Reader may be pleaſed to take notice, that in both the *Moods* of the Datapurall *Figure*, I am in ſome meaſure neceſſitated for the better order ſake, to couch two precepts, or documents, for the *Faciendas* thereof, and to premiſe that one concerning the three *Legs* given, before I make any mention of the maine *Reſolver*, whereupon both the foreſaid *Moods* are founded, to which *Reſolver*, becauſe of both their dependences on it, I have allowed here in the *Gloſſe*, the ſame middle place, which it poſſeſſeth in the Table of my *Triſſotetras*.

The precept of *Nerelema* is *Halbaſalzes* * Ad* Ab* *Sadſabreregalsbis Ir*: that is to ſay, for the finding out of an *Angle* when the three *Legs* are given, as ſoone as we have conſtituted the *ſuſtentative Leg* of that *Angle* a *Baſe*, the *halfe* thereof muſt be taken, and to that *halfe* we muſt *adde halfe* the *difference* of the other two *Legs*, and likewiſe from that *halfe ſubtract* the *half difference* of the foreſaid two *Legs*, then the *ſumme* and the *reſidue* being two *Arches*, we muſt, to the Logarithms of the *Sine* of the *ſumme*, and *Sine* of the *Remainer*, joyne the Logarithms of the Arithmeticall complements of the *Sines* of the *ſides*, which are the refined *Antiſecants* of the ſaid *Legs*, and halfe that ſumme will afford us the Logarithm of the *Sine* of an *Arch*, which *doubled*, is the verticall *Angle*, we demand; for out of its Reſolver, *Parſes—Powto—Parſadſab* ☞ *Powſalvertir*, is the Analogy of the former work made cleare, the *Theorem* being, *As the Oblong or Parallelogram contained under the Sines of the Legs, to the ſquare, power or quadrat of the totall Sine: ſo the Rectangle, or Oblong made of the right Sines of the* ſum, *and* difference *of the halfe Baſe, and difference of the Legs, to the ſquare of the right Sine of halfe the verticall Angle.*

The reaſon hereof will be manifeſt enough to the induſtrious Reader,

Reader, if when by a peculiar *Diagram*, of whoſe equiangular Triangles the foreſaid *Sines* and *differences* are made the conſtitutive *ſides*, he hath evinced their Analogy to one another, he be then pleaſed to perpend, how, in two rowes of proportionall numbers, the *products* ariſing of the homologall *roots*, are in the ſame proportion amongſt themſelves, that the ſaid *roots* towards one another are; wherewithall if he doe conſider, how the *halfs* muſt needs keep the ſame proportion that their *wholes*; and then, in the work it ſelfe of collationing ſeverall orders of proportionall termes, both ſingle and compound, be carefull to daſh out a *divider* againſt a *multiplyer*, and afterwards proceed in all the reſt, according to the ordinary rules of Æquation, and Analogy, he cannot chooſe but extricat himſelfe with eaſe forth of all the windings of this elaboured propoſition.

Upon this *Theorem* (as I have told you) dependeth likewiſe the *Document* for the *faciendum* of *Ralamane*, which is the ſecond *Mood* of *Datapura*, and the laſt of the Monurgetick Loxogonoſphericals, as is pointed at by *Nera* the Directory therof. This *Mood Ralamane* comprehendeth all thoſe Loxogonoſphericall Problems, wherein the three *Angles* being given, a ſide is demanded. And by its Reſolver, *Parſes—Powto—Parſadſab—Powſalvertir*, according to the peculiar precept of this *Mood Kourbfaſines (Ereled) Konfbraxypopyx*, ſheweth, that if we take the complement to a ſemicircle of the *Angle* oppoſite to the *ſide* required, which for diſtinction ſake we doe here call the *Baſe*; and frame, of the foreſaid complement to a ſemicircle, a ſecond *Baſe* for the fabrick of a new Triangle, whoſe other two *ſides* have the graduall meaſure of the former Triangles other two *Angles*: (and ſo the three *Angles* being converted into *ſides*) the complement to a Semicircle of the new *Verticall*, or *Angle* oppoſite to the new *Baſe*, will be the meaſure of the true *Baſe* or *Leg* required, and the *Angle* inſident on the right end of the new *Baſe* in the ſecond Triangle, falleth to be the *ſide* conterminall with the left end of the true *Baſe* in the firſt Triangle, and the *Angle* adjoyning the left end of the falſe *Baſe* in the ſecond Triangle, becomes the *ſide* adjacent to the right end of the old *Baſe* in the firſt Triangle. So that thus by the *Angles* all and each of the *ſides* are found out, all which works are to be performed by the preceding

*Mood*, upon the *Theorem*, whereof the reaſon of all theſe operations doth depend.

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# The Diſergetick Loxogonoſphericals are grounded on foure Axioms, *viz.*

1. *Nabadproſver.* 2. *Naverprortes*, *Siubprortab*, and *Niubprodneſver*: the foure Directories whereof, each in order to its owne *Axiome*, are *Alama*, *Allera*, *Ammena*, and *Ennerra*.

The firſt *Axiome* is, *Nabadproſver*, that is, In Obliquangular Sphericals, if a Perpendicular be demitted from the verticall Angle to the oppoſite ſide, continued if need be, *The Sines complements of the Angles at the Baſe, will be directly proportionall to the Sines of the verticall Angles*, and contrary: the reaſon hereof is inferred out of the proportion, which the Sines of Angles, ſubſterned by Perpendiculars, have to the Sines of the ſaid perpendiculars, ſo that they belong to the Arches of great Circles, concurring in the ſame point, and that from ſome point of the one, they be let fall on the other Arches; which proportion of the Sines of the ſaid Perpendiculars, to the Sines of the Angles ſubtended by them, floweth immediatly from the *proportion, which* (in ſeverall Orthogonoſphericals, having the ſame acute Angle at the Baſe) *is betwixt the Sines of the Hypotenuſas, and the Sines of the perpendiculars*; the demonſtration whereof is plainly ſet downe in my *Gloſſe* on *Suproſca*, the firſt generall *Axiome* of the Sphericals, of which this *Axiome* of *Nabadproſver* is a conſectary.

The Directory of this *Axiome* is *Alama*, which ſheweth, that the *Moods* of *Alamebna* and *Amanepra* are grounded on it.

The ſecond Diſergetick Axiome is *Naverprortes*, that is to ſay, *the Sines complements of the verticall Angles, in obliquangular Triangles* (a Perpendicular being let fall from the double verticall on the oppoſite ſide) *are reciprocally proportionall to the Tangents of the ſides*: the reaſon hereof proceedeth from *Sbaprotca*, the ſecond generall Axiome of the Sphericals; according to which, if we doe but regulate, after the cuſtomary Analogicall manner, two quaternaries

ternaries of proportionals of the former Sines complements, and Tangents propoſed, we will find by the extremes alone (excluding all the intermediate termes) that *the Sines complements of the verticall Angles (both forwardly, and inverſedly) are reciprocally proportioned to the Tangents of the ſides*, and contrariwiſe from the Tangents, to the Sines. The Directory of this Axiome is *Allera*, which evidenceth, that the *Moods* of *Allamebne*, and *Erelomab* depend upon it.

The third Diſergetick Axiome is *Siubprortab*, that is to ſay, that *in Obliquangular Sphericals* (if a perpendicular be drawne from the verticall Angle unto the oppoſite ſide, continued if need be) *the Sines of the ſegments of the Baſe, are reciprocally proportionall to the Tangents of the Angles conterminate at the Baſe, and contrary:* the proofe of this, as well as that of the former conſectary dependeth on *Sbaprotca*, the ſecond generall Axiome of the Sphericals, according to which, if we ſo Diagrammatiſe an Amblygonoſphericall Triangle, by Quadranting the Perpendicular, and all the ſides, and deſcribing from the Baſangulary points two Quadrantall Arches, till we hit upon two rowes of proportionall *Sines* of *Baſes* to *Tangents* of *Perpendiculars*, then ſhall we be ſure (if we exclude the intermediate termes) to fall upon a reciprocall *Analogy* of *Sines*, and *Tangents*, which alternatly changed, will afford the reciprocall proportion of the *Sines* of the *Segments* of the *Baſe*, to the *Tangents* of the *Angles* conterminat thereat, the thing required.

The Directory of this Axiome is *Ammena*, which certifieth that *Ammanepreb* and *Enerablo* are founded thereon.

The fourth and laſt Diſergetick Axiome is *Niubprodneſver*, that is to ſay, that in all *Loxogonoſphericals* (*where the Cathetus is regularly demitted*) *the Sines complements of the Segments of the Baſe, are directly proportionall to the Sines complements of the ſides of the verticall Angles, and contrary.* The reaſon hereof is made manifeſt, by the proportion that is betwixt the Sines of Angles, ſubtended by Perpendiculars and the Sines of theſe Perpendiculars; out of which we collation ſeverall proportions, till, both forwardly and inverſedly, we pitch at laſt upon the direct proportion required.

The Directory of this Axiome is *Ennerra*, which declareth that *Ennerable*, and *Errelome* are its dependents.

# Of the Diſergetick Loxogonoſpherical there be in all foure *Figures*; two *Angulary*, and two *Laterall*.

THe two *Angulary* are *Ahalebmane* and *Ahamepnare*: The two laterall are *Ehenabrole* and *Eheromabne*.

The firſt Angulary Diſergetick Loxogonoſpherical Figure, *Ahalebmane*, comprehendeth all thoſe Problems, wherein two *Angles* being given with a *ſide* betweene, either the third *Angle*, or an oppoſite *ſide* is demanded; and accordingly hath two *Moods*, the firſt whereof is *Alamebna*, and the ſecond *Allamebne*.

*Alamebna* concerneth all thoſe Loxogonoſpherical Diſergetick Problems, wherein two *Angles* being propoſed, with an interjacent *ſide*, the third *Angle* is required; which *Angle*, according to the ſeverall *Caſes* of this Mood, is alwayes one of the *Angles* at the *Baſe*, that is to ſay (in the termes of my *Triſſotetras*) a prime, or next *oppoſite*, or at leaſt one of the *co-oppoſites*, to the Perpendicular to be demitted. And therefore, conforme to the nature of the *Caſe* of the *Datoquære* in hand, and that it may the more conveniently fall within the compaſſe of the Axiome of *Nabadproſver*, an *Angle* by the firſt operation of this *Diſergetick* is to be found out, which muſt either be a *double verticall*, a *verticall* in the little *rectangle*, or a *verticall*, or *co-verticall* (as ſometimes I call it) in one of the *correctangles*.

Thus much I have thought fit to premiſe of the *prænoſcendum* of this *Mood*, before I come to its *Cathetotheſis*; becauſe, in my *Triſſotetrall* Table, to avoid the confuſion of *homogeneall* termes (though the order of doctrine would ſeeme to require another Method) the firſt and prime Orthogonoſpherical work is totally unfolded, before I ſpeak any thing of the variety of the Perpendiculars demiſſion, to which, owing its rectangularity, it thereby obtaineth an infallible progreſſe to the *quæſitum*: but, ſeeing in the *Gloſſe* I am not to aſtrict my ſelf to ſo little bounds, as in my Table, I will obſerve the order that is moſt expedient; and, before the reſolution of any operation in this Mood, deduce the diverſity of the *Perpendiculars* proſilience in the ſeverall *Caſes* thereof.

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Let the Reader then be pleaſed to conſider, that the generall Maxim for the *Cathetotheſis* of this *Mood* is *Cafregpiq*, the meaning whereof is, that, whether the ſide whereon the Perpendicular is demitted be increaſed or not, that is to ſay, whether the Perpendicular fall outwardly, or inwardly, it muſt fall from the extremity of the *given ſide*, and oppoſite to the *Angle required*: however it is to be remarked, that in this *Mood*, whatever be the affection of the Angles (unleſſe they be all three alike) the Perpendicular may fall outwardly.

The generall maxim for the *Cathetotheſis* of this *Mood*, as well as for that of all the reſt, is divided into foure *Tenets*, according to the number of the *Caſes* of every *Mood*.

Here muſt I admoniſh the Reader, that he ſtartle not at the mentioning of foure eſpeciall *Cathetothetick Tenets*, and foure ſeverall *Caſes* belonging to each *Diſergetick Mood*, ſeeing, to the moſt obſervant eye, there be but three of either perceptible in my *Triſſotetras*; for, the fourth both *Tenet* and *Caſe* being the ſame by way of expreſſion in all the *Moods*, and being fully reſolved by the third *Caſe* of every *Mood*, it ſhall ſuffice to ſpeak thereof here once for all: The *Tenet* of this common *Caſe* is *Simomatin*, that is to ſay, when all the three Angles in any of thoſe *Diſergeticks* are of the ſame affection, either all *acute*, or all *oblique*, the Perpendicular falleth inwardly, whether the double verticall be an Angle given, an Angle demanded, or neither. Yet here it is to be conſidered, that ſeeing Triangles may be either calculated by their reall and naturall, or by their circular parts, or by both together, and that for the more facility we oftentimes, inſtead of the propoſed Triangle, reſolve its oppoſite; it is not the reall and given Triangle, that in this caſe we ſo much take notice of, as of its reſolvable, and equivalent, the oppoſite Triangle: as for example, If a Sphericall Triangle, with two obtuſe Angles, and one acute, be given you to reſolve, it will fall within the compaſſe of *Simomatin*; becauſe its oppoſite Sphericall is ſimply acuteangled: and alſo if you be deſired to calculate a Spherical Triangle with two acute Angles, and one obtuſe, it will likewiſe fall within the reach of the ſame *Caſe*; becauſe its oppoſite Spherical is ſimply obtuſangled. The reaſon of both the premiſſes is from the equality of the *oppoſite Angles* of concurring *Quadrants*, which

which that they are equall, no man needs to doubt, that will take the paines to let fall a Perpendicular from the middle of the one *Quadrant* upon the other; for ſo there will be two Triangles made equilaterall: and ſeeing it is an univerſally received truth, that *equall ſides ſuſtaine equall Angles*, the identitie of the Perpendicular in both the foreſaid Triangles, muſt needs manifeſt the equality of the two oppoſite Angles.

I have beene the ampler in the elucidating of this *Caſe*, that, it over-running all the *Moods* of the Diſergetick Loxogonoſphericals, the Reader, in what *Mood* or *Datoquere* ſoever he pleaſe to reſolve this foreſaid *Caſe*, may for that purpoſe to this place have recourſe; to the which, without any further intended reiteration of this Tenet, I doe heartily remit him.

The firſt eſpeciall *Tenet* of the generall *Maxim* of the *Cathetotheſis* of this *Mood* is *Daſimforaug*, that is, *When the given Angles are of the ſame nature, but different from the required, the Perpendicular falleth outwardly, and the firſt verticall is a given Angle*: the ſecond *Tenet* belonging to the ſecond *Caſe* of this *Mood* is *Dadiſforeug*, that is, *When the propoſed Angles are of different affections, the Perpendicular is externally demitted, and one of the given Angles is a ſecond verticall*: Yet this diſcrepance is to be obſerved betweene the externall proſiliencie of the Perpendicular Arch in this *Caſe*, and that other of the former; that in the former, it is no matter from which of the ends of the propoſed ſide, the Perpendicular be let fall upon one of the comprehending Legs of the Angle required, which Leg muſt be increaſed; for it is a generall Notandum, that *the ſuſtentative Leg of a perpendiculars exterior demiſſion muſt alwayes be continued*: but in this *Caſe*, the outward falling of the Perpendicular is onely from one extremity of the given ſide; for, if it be demitted likewiſe from the other end, it falleth then inwardly, and ſo produceth the third *Tenet* of this *Mood*, which is *Dadiſgatin*, that is, *If the given Angles be of a different quality, and that the Perpendicular be internally demitted, the double verticall is one of the propoſed Angles.*

The nature of the Perpendiculars demiſſion in all the *Caſes* of this *Mood* being thus to the full explained, we may without impediment proceed to the performance of all the Orthogonoſphericall operations, each in its owne order thereto belonging.

To

To begin therefore at the firſt, whoſe *quæſitum* (as I have told you already) is a *verticall Angle*, we muſt know, ſeeing the work is Orthogonoſpherically to be performed, that the forementioned *prænoſcendum* cannot be obtained without the help of one of the ſixteen *Datoquæres*; and therefore in my *Triſſoſetras* (conſidering the nature of what is given, and asked in the *Caſes* of this *Mood*) I have appointed *Upalam* to be the ſubſervient of its *prænoſcendum*; for, by the *Reſolver* thereof, *To—Tag—Nu☞☜ Mir*, (the ſubtendent and an Angle being given; for one of the given ſides of every Loxogonoſpherical, if the Perpendicular be rightly demitted, becomes a ſubtendent, and ſometimes two given ſides are ſubtendents both) we frame theſe three peculiar Problems, for the three *prænoſcendas*; to wit, *Utopat*, for the *double verticall*, by the meanes of the great *Subtendent* ſide, and the *prime oppoſite Angle*: ſecondly, *Udobaud*, for obtaining of the *firſt verticall* in the little rectangle, by vertue of the *leſſer ſubtendent* in the ſame rectangle, and the *next oppoſite Angle*: laſtly, *Uthophauth*, for the *firſt Co-verticall*, by meanes of the *firſt Co ſubtendent*, and *firſt cooppoſite Angle*: all which is at large ſet downe in the firſt *partition* of *Alamebna* in my Table.

The firſt and chiefe operation being thus perfected, the verticall Angles ſo found out muſt concurre with each its correſpondent oppoſite for the obtaining of the Perpendicular, neceſſary for the accompliſhment of the ſecond operation in every one of the *Caſes* of the foreſaid *Mood*; to which effect *Amaner* is made the *Subſervient*, by whoſe Reſolver *Say—Nag—Tw☞☜ Nyr*, theſe three Datoquæres, *Opatca*, *Obamca*, and *Ophauthca* come to light, and is manifeſtly ſhown how, by any paire of three ſeverall couples of different Angles, the Perpendicular is acquirable.

Now, though of this work (as it is a ſingle one) no more then of the other ſucceeding it in the ſame Mood, nor of the laſt two in any of the *Diſergeticks* in their full Analogy, I doe not make any mention at all in my *Table*; but, after the couching of the firſt operation for the Prænoſcendas, ſupply the roomes of the other two, with an equivalent row of proportionals out of them ſpecified, for attaining to the knowledge of the maine *quæſitum*: yet in this *Comment* upon that *Table*, for the more perſpicuities ſake, and that the Reader may as well know, what way the rule

is

is made, as how thereby a demand is to be found out, I have thought fit to expatiat my ſelfe for his ſatisfaction on each operation apart, and Analytically to diſplay in the *gloſſe*, what is compounded in the *Triſſotetras*.

And therefore, according to that preſcribed method, to proceed in this *Mood*, the perpendicular by the ſecond operation being already obtained, it is requiſite for the promoving of a third work, that the ſaid Perpendicular be made to joyne with *the ſecond verticaline, the double vertical, and ſecond co-vertical* conforme to the quality of the three Caſes, thereby to obtaine the Angles *at the Baſe*, for the which all theſe operations have beene ſet on foot; to wit, *the next cathetoppoſite*, (whoſe complement to a Semicircle is alwayes the Angle required) *the prime cathetoppoſite*, and *the ſecond Cocathetoppoſite*: for the proſecuting of this laſt work, *Edoman* is the ſubſervient, by whoſe Reſolver, *To—Neg—Sa☞Nir*, we are inſtructed how to regulate the Problemets of *Catheudob*, *Cathatop*, and *Catheuthops*.

Now theſe two laſt operations being thus made patent in their ſeverall ſtructures, it is not amiſſe that we ponder how appoſitely they may be conflated into one, to the end, that the verity of all the finall Reſolvers of the Diſergeticks in my *Triſſotetras* (which are all and each of them compoſed of the ingredient termes of two different works) may be the more evidently knowne, and obvious to the reach of any ordinary capacity, for the performance hereof, the Reſolvers of theſe two operations are to be ſaid before us, *Say—Nag—To☞Nyr*, and *To—Neg—Sa☞Nir*: and, ſeeing out of both theſe orders of proportionals, there muſt reſult but one, it is to be conſidered, which be the foure *ejectitious* termes, and which thoſe foure we ſhould *reſerve* for the Analogy required; all which, that it may be the better underſtood by the induſtrious Reader, I will interpret the Reſolvers ſo farre forth as is requiſite: and therefore *Say—Nag—To☞Nyr*, being, *As the Sine of one of the Angles at the Baſe, or Cathetoppoſite, is to the Sine complement of a vertical: ſo the Radius to the Sine complement of the Perpendicular:* And the other, *To—Neg—Sa☞Nir*, being, *As the Radius, to the Sine complement of the Perpendicular: ſo the Sine of a vertical, to the Sine complement of a Cathetoppoſite or Angle at the Baſe*; it is perceivable enough how

how both the *Radius*, and the *Perpendicular* are in both the rows: nor can it well eſcape the knowledge of one never ſo little verſed in the elements of Arithmetick, that the Perpendiculars being the fourth terme in the firſt order of proportionals, is nothing elſe but that it is the quotient of the product of the middle termes, divided by the firſt, or *Logarithmically* the remainder of the firſt termes abſtraction from the ſumme of the middle two; ſo that the whole power thereof is incloſed in theſe three termes, whereby it is moſt evident, that with what terme ſoever the foreſaid Perpendicular be employed to concurre in operation, the ſame effect will be produced by the concurrence of its ingredients with the ſaid terme, and therefore in the ſecond row of proportionals, where it is made uſe of for a fellow multiplyer with the third terme to produce a *factus*, which divided may *quote* the maine *quæſitum*, or *Logarithmically* to joyne with the third terme, for the ſumming of an Aggregat from which the firſt terme being abſtracted, may reſiduat the terme demanded, it is all one, whether the work be performed by it ſelfe, or by its equivalent, *viz.* the three firſt termes of the firſt order of proportionals, in whoſe potentia it is: whereupon the fourth terme in the ſecond row being that, for the obtaining whereof, both the Analogies are made, we need not waſte any labour about the finding out of the Perpendicular (though a ſubſervient to the chiefe quæſitum) but leaving roome for it in both the rows, that the equipollencie of its conflaters may the better appeare, go on in work without it, and, by the meanes of its conſtructive parts, with as much certainty effectuat the ſame deſigne.

Thus may you ſee then how the eight termes of the forementioned Reſolvers, are reduced unto ſix; but there remaining yet two more to be ejected, that both the orders may be brought unto a compound row of foure proportionals: let us conſider the *Radius*, which, being in both the rows as I have once told you already, may peradventure, without any prejudice to the work, be ſpared out of both. Thus much thereof to any is perceiveable, that in the *firſt* Reſolver, it is the *third* proportionall; and in the *ſecond*, the *firſt*, and conſequently a *multiplyer* in the *one*, and in the *other* a *divider*: or *Logarithmically* in the *ſecond* a *ſubtracter*, and in the *firſt* an *adder*: now it being well known that *diviſion* over-

throws the ſtructure of *multiplication*, and that what is made up by *addition*, is by *ſubtraction* caſt down; we need not undergoe the laboriouſneſſe of ſuch a Penelopæan task, and by the *diviſion* and *abſtraction* of what we did *adde* and *multiply*, weave and unweave, build up, and throw downe the ſelf ſame thing: but chooſe rather (ſeeing the *Radius* undoeth in the *one*, what it doth in the *other* (which ineffect is to doe nothing at all) to daſh the one againſt the other, and race it out of both) then idely to expend time, and have the proportion peſtred with unneceſſarie termes. Thus from thoſe two reſolvers, foure termes being with reaſon ejected, we muſt, for the finding out of the laſt in the ſecond Reſolver, effectuat as much by *three*, as formerly was on *ſeven* incumbent, which *three* being the firſt, and ſecond termes in the firſt row of proportionalls; and the third in the ſecond, the two Reſolvers *Say—Nag—To ☞ Nyr*, & *To—Neg—Sa ☞ nir* are comprehended by this one *Say—Nag—Sa—☞ Nir*, that is, *As the ſine of one verticall to the Antiſine of an oppoſite; ſo the ſine of another verticall to the Antiſine of another oppoſite*: and though the ſecond Reſolver doth import, that this other oppoſite is to be found out by the Antiſine of the perpendicular, and ſine of a ſecondarie verticall, yet doth it in nothing evince the coincidence of the two operations in one; becauſe the firſt two termes of the Reſultative Analogie, doe adæquatly ſtand for the perpendicular, which I have proved already, and therefore theſe *two* in their proper places co-working with the *third* terme, according to the rule of proportion, have the ſelfe ſame influence, that the Perpendicular ſo ſeconded, hath upon the *operatum*.

Now, to contract the generality of this finall Reſolver, *Say—Nag—Sa ☞ Nir*, to all the particular *Caſes* of this *Mood*, we muſt ſay, *When the given Angles are of the ſame affection, and the required diverſe*, as in *Daſimforaug*, the firſt caſe, *Sat—Nop—Seud ☞ Nob*Kir*, that is, *As the Sine of the double verticall to the Antiſine of the prime Cathetoppoſite: ſo the Sine of the ſecond verticaline* (or verticall in the leſſer rectangle) *to the Antiſine of the next Cathetoppoſite*, whoſe complement to a ſemicircle is the *Angle* required.

But when, the affection of the given Angles being different, the perpendicular is made to fall without, as in *Dadiforeug*, the ſecond *Caſe* of this *Mood*, the Reſolver thereof is particulariſed thus,

thus, *Saud—Nob—Sat☞Nop * Ir*, that is, *As the Sine of the first verticaline* (or verticall in the rectanglet,) *to the Sine complement of the neareſt Cathetoppoſite: ſo the Sine of the double verticall, to the Sine complement of the prime Cathetoppoſite*, which is the Angle required.

And laſtly, if with the different qualities of the *given* and *demanded* Angles, the Perpendicular be let fall within, as in *Dadiſgatin*, the third *Caſe* of this *Mood*, then is the finall Reſolver to be determined thus, *Sauth—Noph—Seuth☞Nops * Ir*, that is, *As the Sine of the firſt coverticall, to the Co-ſine of the firſt Co-oppoſite: ſo is the Sine of the ſecond coverticall, to the Co-ſine of the ſecond Co-oppoſite* which is the *Angle* required.

The originall reaſon of all theſe operations is grounded on the Axiome of *Nabadproſver*, as the firſt ſyllable of its Directory *Alama* giveth us to underſtand, which we may eaſily perceive by the Analogy, that is onely amongſt the *Angles* without any intermixture of *ſides* in the termes of the proportion.

The ſecond *Mood* of the firſt Angulary *Figure* (that is to ſay, the firſt two termes of whoſe datas are Angles) is *Allamebne*, which comprehendeth all thoſe Diſergetick queſtions, wherein two *Angles* being given and a *ſide* betweene, one of the other *ſides* is demanded, which *ſide* (the perpendicular being let fall) is alwayes one of the *ſecond Subtendents*, viz. in the firſt Caſe a *ſecond Subtendent* of the leſſer Triangle, in the ſecond a *ſecond Subtendent* in the great rectangle, and in the laſt a *ſecond Co-ſubtendent*.

To the knowledge of all theſe, that we may the more eaſily attaine, we muſt conſider the generall maxim of the *Cathetotheſis* of this *Mood*, which is *Cafyxegeq*, that is to ſay, that *in all the Caſes of Allamebne the Perpendicular falleth from the ſide required, and from that point thereof, where it conterminats with the given ſide upon the third ſide, continued if need be*; and according to the variety of the *ſecond ſubtendent*, which is the ſide demanded, there be theſe three eſpeciall *Tenets* of this generall *Maxim*, to wit, *Daſimforauxy*, *Dadiſcracſoreug*, and *Dadiſcramgatin*.

*Daſimforauxy*, the firſt eſpeciall *Tenet* of the generall *Maxim* of the *Cathetotheſis* of this *Mood* ſheweth, that, *when the propoſed Angles are of the ſame quality and homogeneall, the Perpendicular falleth externally, and the firſt verticall is one of the given*

*Angles*, and annexed to the required ſide.

The ſecond Tenet, *Dadiſcracforeug*, which pertaineth to the ſecond *Caſe* of this *Mood*, ſheweth, that *when the given Angles are of a diſcrepant nature, and heterogeneall, and that the concurſe of the propoſed and required ſides is at an acute Angle, that then the Perpendicular muſt be demitted outwardly, and one of the propoſed Angles becomes a ſecond verticall.*

The third *Tenet* is *Dadiſcramgatin*, whereby we learne, that if *with the various affection of the Angles given, the concurſe* (mentioned in the preceding Tenet) *be at an obtuſe Angle, the Perpendicular falleth inwardly, and that one of the foreſaid Angles is a double verticall.* This is the onely *Caſe* of *Allamebne*, wherein the Perpendicular is demitted inwardly, ſave when the three Angles are qualified all alike, of which *Caſe*, becauſe it falleth in all the Moods of the Loxogonoſpherical Diſergeticks, and that in *Alamebna* I have ſpoke at large thereof, I ſhall not need (I hope) to make any more mention hereafter.

Having thus unfolded the myſteries of the Perpendiculars demiſſion in all the *Caſes* of this *Mood* (as I muſt doe in all thoſe of every one of the other Loxogonoſpherical Diſergeticks; becauſe ſuch Obliquangulars, till they be reduced to a rectangularity (which without the Perpendicular is not performable) can never Logarithmically be reſolved) I may ſafely go on, without any let to the Reader, to the three ſeverall Orthogonoſpherical operations thereof, as they ſtand in order.

The *quæſitas* of the firſt operation, which are alwayes the *prænoſcendas* of the *Mood*, are in this *Mood* the ſame that they were in the laſt, to wit, the double *verticall, the firſt verticaline, and the firſt coverticall:* and are likewiſe to be found out by the ſame Datas both of *ſide*, and *Angle* here, that they were in the former *Mood*; that is, for the *ſide*, by the *firſt and great Subtendent: the firſt but little Subtendent: and the firſt Co-ſubtendent:* and for the *Angle*, by the *prime Cathetoppoſite, the neareſt Cathetoppoſite, and the firſt Co-cathetoppoſite:* ſo that the *Datoquare* ſounding thus, *the Subtendent, and an Oblique Angle being given, to find the other Oblique*, the Subſervient of this Computation muſt needs be *Ilpatam*, and its Reſolver, *To—Tag—Nuſ☞Mir*, which ſheweth, that the ſubducing of the Logarithm of the Radius from the

ſumme

ſumme of the Logarithms of the Sine complement of one of the firſt Subtendents, and Tangent of one of the Angles at the Baſe, reſiduats the Logarithm of the Tangent complement of one of the verticals required, and conſequently involveth within ſo much generality the particular reſolutions of the *Sub-problems* of *Upalam*, viz. *Utopat*, *Vdobaud*, and *Uthophauth*, diverſified thus according to the variety of their prænoſcendas, whereon, to ſpeak ingenuouſly, I intend to inſiſt no longer; for, beſides that the peculiar enodation of all the three apart is clearly ſet downe in my *gloſſe* on the laſt *Mood*, they are in both the firſt *partitions* of the *Moods* of *Ahalebmane* to the full expreſſed in the Table of my *Triſſotetras*.

The verticall Angles, according to the diverſity of the three *Caſes* being by the foreſaid Datas thus obtained, muſt concurre with each its correſpondent firſt Subtendent (notified by the Characteriſticks of 7. 8. 9.) for finding out of the perpendicular, requiſite for the performance of the ſecond work in every one of the *Caſes* of this *Mood*. And to this effect *Ubamen* is made the Subſervient, by whoſe Reſolver *Nag—Mu—Torp Myr*, theſe three Problems, *Utatatca*, *Vdaudca*, and *Uthauthca*, are made manifeſt, and the ſame *quæſitum* attained unto by the Datas of three ſeverall Subtendents, and verticals.

The Perpendicular being thus found out, muſt, for the furtherance of the third operation, joyne with the *ſecond verticaline*, the *double verticall*, and *ſecond co-verticall*, according to the nature of the *Caſe* in queſtion, (the Datas being the ſame with thoſe of the third work of the laſt *Mood*) thereby to attaine unto the knowledge of the ſecond *little Subtendent*, the ſecond *great Subtendent*, and the ſecond *Co-ſubtendent*, the which are all the maine *quæſitas* of this *Mood*: To the performance of this laſt operation, *Etalum* is the ſubſervient, whoſe Reſolver *Torp—Me—Nag Mur*, teacheth us how to deale with the *under datoquæres* of *Cathendwd*, *Cathstwt*, and *Catheuthwth*.

Now, the coaleſcencie of theſe laſt two operations in one, proceeding from the caſting out of the Radius in both the orders of propottionals, and leaing roome for the perpendicular, without taking the paines to know its value, as hath beene ſhewne already in the firſt *Mood* of the ſame *Figure*; it cannot be much amiſſe

amiſſe in this place to give a further illuſtration thereof, and make the Reader, by an Arithmeticall demonſtration, feele (as it were) how palpable the truth is of compacting eight proportionals into foure; let there be then theſe two orders of numbers, 4—6—8☞12. and 8—12.—14☞21. Where, we may ſuppoſe eight to be the Radius, and twelve the Perpendicular (for ſuch like ſuppoſitions can inferre no great abſurdity) and then let us conſider how thoſe termes doe beare to one another, eſpecially the 12. and 8. which, by poſſeſſing foure places, make up halfe the number of the proportionals. Firſt, we ſee that twelve in the firſt row, is nothing elſe but the reſult of the product of 6. in 8. divided by 4. And ſecondly, that 8. in the ſecond row, caſteth downe, by its diviſion, whatſoever by its multiplication it builded up in the firſt; upon which obſervations we may ground theſe Sequels, that 12. may be ſafely left out, both in the fourth, and ſixth place, taking inſtead of it the number of 4. 6. and 8. in whoſe *potentia* it is: and next 8. undoing in one place, what it doth in another, may with greater eaſe void them both. So that by this abbreviated way of Analogiſing, 4. and 6. alone in their due order before 14. which is the third terme of the ſecond row, conduce as much to the obtaining of the fourth, or if you will eighth proportionall 21. as if the other foure termes of the two eights, and twelves, were concurrent with it. How plaine all this is, no queſtion needs to be made, and therefore, to returne to our Reſolvers (for the explicating whereof, we thought good to make this digreſſion) we muſt underſtand that the finall Reſolver, (in its generall expreſſion) made out of them (they being as they are materially diſplayed, *Nag-Mu-Torp*☞*Myr*, & *Torp-Me-Nag*☞*Mur*) is no other then *Nag-Mu-Na*☞*Mur*, that is, *As the Sine complement of one verticall is to the Tangent complement of a Subtendent: So the Sine complement of another verticall, to the Tangent complement of another Subtendent:* and Analytically to trace the running of this operation, even to the ſource from whence it flowes, by foyſting in the Perpendicular, and Radius, we may bring it to the conſiſtence of the former two ſubordinate Reſolvers, whereof the firſt is, *As the Sine complement of a firſt, or a double verticall, to the Tangent complement of a firſt Subtendent: ſo the Radius to the Tangent complement of the Perpendicular*; and the ſecond, *As the*

*Radius,*

*Radius, to the Tangent complement of the Perpendicular : ſo the Sine complement of a ſecond, or a double verticall, to the Tangent complement of a ſecond Subtendent,* which is the ſide required, and the fourth proportionall of *Nag—Mu—Na☞Mur.* Whoſe generality is to be contracted to every one of the three *Caſes* of this *Mood* thus: If both the Angles given be of the ſame nature, they being the firſt verticals, from which the *Cathetus* fals on either ſide, increaſed according to the demand of the ſide, as in the firſt Caſe, *Daſimforauxy,* we muſt particulariſe the common Reſolver, in this manner, *Nat—Mut—Neud☞Nwd * Yr,* that is, *As the Antiſine of the double verticall, is to the Antitangent of the firſt, and great Subtendent: ſo the Antiſine of the ſecond verticall in the leſſer rectangle, to the Antitangent of the ſecond Subtendent in the ſame little rectangle,* which Subtendent is the ſide required. For the ſecond *Caſe* of this *Mood,* viz. *Dadiſcraeforeug,* we muſt ſay, *Naud—Mud—Nat☞Mwt * Yr,* that is, *As the Sine complement of the firſt and little verticall to the Tangent complement of the firſt, and little Subtendent: ſo the Sine complement of the double verticall, to the Tangent complement of the ſecond and great ſubtendent.* And laſtly, for the third Caſe *Dadiſcramgatin,* the finall Reſolver is determinated thus, *Nauth—Muth—Neuth☞Mwth * Yr,* that is, *As the Co-ſine of the firſt Co-verticall, is to the Co-tangent of the firſt Co-ſubtendent : ſo the Co-ſine of the ſecond Co-verticall, to the Co-tangent of the ſecond Co-ſubtendent,* which is the *ſide* in this third *Caſe* required.

The truth of all theſe operations is grounded on the Axiome of *Naverprortes,* as we are certified by the firſt ſyllable of its Directory *Allera,* which we may perceive by the direct Analogy that is betweene the Sines complements of the verticall Angles, and the Tangents complements, (and conſequently reciprocall twixt them and the Tangents) of the verticall ſides, which in this *Mood* are alwayes ſecond Subtendents.

The ſecond Diſergetick, and Angulary *Figure*, is *Ahamepnare*, which embraceth all thoſe Obliquangularie Sphericals, wherein *two Angles* being given with an *oppoſite ſide,* another *Angle,* or the *ſide interjacent,* is demanded : this *Figure,* conforme to the two ſeverall *Queſitas,* hath two *Moods,* viz. *Amanepra*, and *Ammanepreb.*

The

The firſt *Mood* hereof, which is *Amanepra*, belongeth to all thoſe Loxogonoſpherical queſtions, wherein, *two Angles with an oppoſite ſide being propoſed, the third Angle is required*, which is alwayes a *firſt verticall*, a *ſecond verticall*, or a *firſt co-verticall*: to the notice of all which, that we may with eaſe attaine, the generall *Maxim* of the *Cathetotheſis* of this *Mood* is to be conſidered, which is *Cafriq*, that is to ſay, that in all the *Caſes* of *Amanepra*, the *Perpendicular* falleth from the *Angle* required upon the *ſide* oppoſite to that *Angle*, and terminated by the other two *Angles*, which ſide is to be increaſed, if need be.

Now in regard, that beſides the *Cathetotheſis* of this Mood, and ſome three moe, to wit, all thoſe Loxogonoſpherical wherein the *quæſitum* is either a *partiall verticall*, or *ſegment at the Baſe*, there is a peculiar *Menſurator*, pertaining to every one of the foure, called in my *Triſſotetras* the *plus minus*, becauſe it ſheweth by the ſpecieſes thereof to the *Moods* appropriated, whether the ſumme, or difference of the verticall *Angles*, and ſegments at the *Baſe*, be the *Angle*, or *ſide* required, and ſo clearly leadeth us thorough all the *Caſes* of each of the *Moods*, that either by abſtracting the fourth proportionall from an *Angle* or a *ſegment*, or by abſtracting an *Angle*, or a *ſegment* from it, or laſtly, by joyning it to an *Angle*, or a *ſegment*, with an incredible facility we attaine to the knowledge of the *maine quæſitum*, whether *Angulary*, or *lateraall*. Let the Reader then be pleaſed to know, that the *Menſurator*, or *Plus minus* of this *Mood*, is *Sindifora*, which evidently declareth (as by its repreſentatives in the explanation of the Table is apparent) that, if the demiſſion of the Perpendicular be internall, the ſumme; if *exterior*, the *difference* of the *verticall Angles*, is the *Angle* required.

Seeing thus the notice of the manner of the Perpendiculars falling is ſo neceſſary, it is expedient, for our better information therein, that we ſeverally perpend the three eſpeciall *Tenets* of the generall *Maxim* of the *Cathetotheſis* of this *Mood*, which are *Dadiſſepamforaur Dadiſſexamforeur*, and *Daſimatin*.

*Dadiſſepamforaur*, which is the *Tenet* of the firſt *Caſe*, ſheweth, that when the *Angles* given are of a different nature, and that the propoſed *ſide* is oppoſite to an obtuſe *Angle*, the Perpendicular falleth outwardly, and the firſt verticall is the *Angle* required.

The

The ſecond *Tenet* belonging to the ſecond *Caſe* of this *Mood*, viz. *Dadiſſexamſoreur*, ſheweth, that if the propoſed *Angles* be of diſcrepant affections, and that the *ſide* given be conterminat with an *obtuſe Angle*, the Perpendicular is demitted externally, and the demanded *Angle* is a ſecond *verticall*.

The third *Tenet* pertaining to the laſt *Caſe* of this *Mood*, to wit, *Daſimatin*, evidenceth, that if the *Angles* propoſed be of the ſame quality, the Perpendicular falleth interiourly, and the double verticall is the *Angle* required.

Having thus (as I ſuppoſe) hereby evinced every difficulty of the Perpendiculars demiſſion in all the *Caſes* of this *Mood*, I may the more boldly in the interim proceed to the three rectangular works thereto belonging. Now, it being manifeſt that the *Præno ſcendas* of this *Mood*, or the *Queſitas* of the firſt operation thereof, are the ſame with thoſe of the two *Moods* of the firſt Diſergetick Figure, to wit, the *double verticall*, *the firſt verticalne*, and the *firſt co-verticall*; and that, without any alteration at all, they are to be obtained by the ſame *Datas*, both of *ſide*, and *Angle* in this *Mood* of *Amanepra*, that, they were in the former *Moods* of *Alamebna*, and *Allamebne*, without any further ſpecifying what theſe given *ſides*, and *Angles* are (which are to the full expreſſed in the laſt two forementioned *Moods*) I muſt make bold thither to direct you, where you ſhall be ſure alſo to learne all that is neceſſary to know of the *Subſervient* and *Reſolver* of the firſt operation of this *Mood*, both which, to wit, *Upalam* and *To—Tag—Nu—Mir*, are inſeparable dependents on all the Angularie *Præno ſcendas* of the Loxogonoſphericall Diſergeticks: And though within the generality of this *Subſervient* be compreded the peculiar *Problemets* of *Utopat*, *Udobaud*, and *Uthophauth*, which are all three at large couched in the *Triſſotetras* of this *Mood*; yet, becauſe what hath beene already ſaid thereof in the foreſaid Figure, may very well ſuffice for this place, the Readers diligence (I hope) in the turning of a leaf, will ſave me the labour of any further recapitulation.

The Præno ſcendas, or the verticall *Angles*, according to the nature of the *Caſe*, being by the foreſaid *Datas* thus found out, muſt needs joyne with each its correſpondent oppoſite, ſpecified by the characteriſticks of $\pi.c.\varphi.$ for the obtaining of the Perpendi-

pendicular, which in all the reſt of the Diſergetick *Moods*, as well as this, is alwayes the *quæſitum* of the ſecond operation, thorough all the *Caſes* thereof. Of this work *Amaner* is the *ſubſervient*, by whoſe Reſolver, *Say—Nag—To ☞ Nyr*, the three ſub-problems, *Opatca*, *Obandca*, and *Ophanthca*, are made known, and the ſame *quæſitum* attained unto by the *Datas* of three ſeveral both cathetoppoſites, and verticals, it being the only *Mood* which with *Alamebna*, hath a cathetoppoſite and verticall catheteuretick identity. The Perpendicular being thus obtained, is, for the effecting of the third and laſt operation, to concurre with the next *cathetoppoſite*, the prime *cathetoppoſite*, and the ſecond *cocathetoppoſite*, as the *Caſe* requires it, thereby to find out the main *quæſitum*; which in the firſt *Caſe* by abſtracting the fourth proportionall, in the ſecond by abſtracting from the fourth proportionall, and in the third by adding the fourth proportionall to another verticall, is eaſily obtained by thoſe that have the skill to diſcerne which be the greater, or leſſer of two verticals propoſed. To the perfecting of this third work, *Exoman* is the *Subſervient*, whoſe Reſolver *Ne-To-Nag ☞ Sir*, inſtructeth us, how to unfold the peculiar Problems of *Cathobeud*, *Cathopat*, & *Cathopſouth*.

Now, the nature of proportion requiring that of two rowes of proportionals, when the fourth in the firſt order is firſt in the ſecond, that then the multiplyers become dividers, and the dividers multiplyers: as by theſe numbers following you may perceive, *viz.* 2—4—6 ☞ 12. for the firſt row, and 12—4—15 ☞ 5, for the ſecond; of which proportionals, becauſe of the fourth terme in the firſt rowes being firſt in the ſecond, if you turne as many multiplyers into dividers as you can, and (where the identity of a Figure requires it) daſh out a multiplyer againſt a divider, you will find, the two foures by this reaſon being raced out, and the two twelves (becauſe of their being in the power of the three firſt proportionals of the firſt row) likewiſe left out, that this Analogy of 6—2—15 doth the ſame effect, that the former ſeven proportionals, for obtaining of the *quæſitum*, viz. 5. the reaſon whereof is altogether grounded upon the inverſion of a permutat proportion, or the Retrograd Analogy of the alternat termes, whereby the Conſequents are compared to Conſequents, and Antecedents to Antecedents, in the prepoſterous method of beginning

beginning at the ſecond of both the Conſequents and Antecedents, and ending at the firſt: therefore (as I was telling you) the nature of proportion requiring that in ſuch a *Caſe* the multiplyers and dividers be bound to interchange their places, the Reſolvers of the laſt two operations, viz. *Say—Nag—To—Nyr*, and *Ne—To—Nag—Sir*, the firſt whereof being, *As the Sine of a verticall Angle, to the Sine complement of an Angle at the Baſe, or one of the Cathetoppoſites: ſo the Radius to the Sine complement of the Perpendicular:* and the ſecond, *As the Sine complement of the Perpendicular, to the Radius: ſo the Sine complement of one of the Cathetoppoſite Angles, to one of the verticals,* may both of them (according to the former rule) be handſomely compacted in this one Analogy, *Na—Say—Nag—Sir*, that is, *As the Sine complement of an oppoſite is to the Sine of a verticall: ſo the Sine complement of another oppoſite, to the Sine of another verticall.*

This foreſaid generall *Reſolver*, according to the three ſeverall caſes of this *Mood*, is to be ſpecialiſed into ſo many finall Reſolvers; the firſt whereof for *Dadiſſepamforaur*, *Nop—Sat—Nob—Seudſr* At* Aut*ir*, that is, *As the ſine complement of the prime cathetoppoſite, to the ſine of the double verticall: ſo the ſine complement of the neareſt cathetoppoſite, to the ſine of the ſecond verticalin; the which ſubtracted from the double verticall, leaveth the firſt and great verticall, which is the Angle required.*

Next, for the ſecond *Caſe* of this *Mood*, *Dadiſſexamforeur*, we muſt make uſe of, *Nob—Saud—Nop—Satfr*, **Aud*Eut*ir*, that is, *As the ſine complement of the next oppoſite, to the ſine of the firſt verticallet: ſo the ſine complement of the prime oppoſite, to the ſine of the double verticall,* from which, *if you deduce the firſt verticalin, there will remaine the ſecond and great verticall for the Angle demanded.*

Laſtly, for the third *Caſe*, *Daſimatin*, we muſt, ſay *Noph—Sauth—Nops—Seuth*jauth*ir*, that is, *As the ſine complement of the firſt co-oppoſite, to the ſine of the firſt co-verticall: ſo the ſine complement of the ſecond co-oppoſite, to the ſine of the ſecond co verticall, which added to the firſt co-verticall, maketh up the Angle we deſire.*

The veritie of all theſe operations is grounded on the Axiome *Nabadproſver*, as the ſecond ſyllable of its directorie *Alama*, giveth us underſtand, and as we may diſcerne more eaſily by the ſameneſſe in ſpecies amongſt the proportionall termes; for they

are all Angles, the firſt, and third being Angles at the Baſe (for theſe are alwaies of the oppoſits) and the ſecond, and fourth termes of the verticall Angles, which verticall Angles in the finall reſolvers of this *Mood*, are according to the foreſaid Axiome, *to the Angles of the Baſe directly proportionall, and contrarily.*

The ſecond *Mood* of the ſecond Angularie *figure* of the Loxogonoſpherical Diſergeticks, named *Ahamepnare* is *Ammanepreb*, which is ſaid of all thoſe obliquangularie problems, wherein *two Angles, and an oppoſite ſide being given, the ſide between is required*, and is alwaies one of the *baſal-ſegments*: to the knowledge whereof, that we may the more eaſily attaine, we muſt conſider the generall *maxime* of the *Cathetotheſis* of this *Mood*, which is *Cafregpagyq*; that is, that the perpendicular falleth ſtill from the given ſide, oppoſite to both the Angles given, and upon the ſide required, continued, if need be, in all and every one of the caſes of *Ammanepreb.*

The *Pluſminus* of this *Mood*, is *Sindiforin*, that is to ſay, *the ſumme of the ſegments of the Baſe, if the perpendicular fall inwardly, and the difference of the Baſes, if exteriorly, is the ſide demanded.*

The perpendiculars demiſſion, being a *Sine quo non* in all diſergetick operations, it will not be amiſſe, that we ponder what the three *ſeverall tenets* are of the *Cathetotheſis* of this *Mood*, and what is meaned by *Dadiſſepamfor*, *Dadiſſexamfor*, and *Daſimin.*

*Dadiſſepamfor*, the *tenet* of the firſt *Caſe* of this *Mood*, ſheweth, that *if the given Angles be of ſeverall natures, and that the propoſed ſide be oppoſite to an obtuſe Angle, the perpendicular falleth externally.*

The ſecond tenet, *Dadiſſexamfor* expreſſeth, that *if the propoſed Angles be different, and that the ſide given be conterminat with the obtuſe Angle, it falleth likewiſe outwardly.*

But *Daſimin*, which is the third *tenet* ſignifieth, that if *the given Angles be of the ſame affection, the falling of the perpendicular is internall.*

This much being premiſed of the perpendicular, we may ſecurely goe on to the orthogonoſpherical works of the Mood; and ſo beginning with the firſt operation, conſider what the *prænoſcendas* are, which are alwaies the *quæſitas* by the firſt operation obtainable, and in this Mood the Baſes of the Triangle; but more particularly to deſcend to the illuſtration of the *Caſes* of *Ammanepreb*,

*manepreb*, the *præno ſcendum* of the firſt *Caſe*, is the *firſt* and *great Baſe*, of the ſecond, the firſt but little *Baſe*, and of the third, the firſt *co-baſe*. Now, though theſe three *præno ſcendas*, be totally different from thoſe of the three former *Moods*, yet are they to be acquired by the ſame, and no other *Datas*; becauſe none of the Angularie *figures* muſt differ from one another in the *Datas* of their *præno ſcendas*, as out of the definition of an Angularie *figure* in the entrie, of the ſecond *Mood* ſet downe, is eaſie to be collected: theſe *Datas* being tendred to us of intermixed circularie parts, that is to ſay, of both ſides and Angles, the ſide being the firſt *ſubtendentall*, or great ſubtendent, the firſt *ſubtendentine*, or little ſubtendent, and the firſt *co-ſubtendent*: and the Angles the *prime cathetoppoſite*, the next *cathetoppoſite*, and the firſt *co-catheoppoſite*; ſo that conſidering what is demanded, and that the *Datoquære* thereof muſt be expreſſed thus, *the hypotenuſa, and an oblique being given, to finde the Ambient conterminate with the propoſed Angle*, we are, for the calculation of this work, neceſſitated to have recourſe to *Vbamen*, which, in the Table of my *Triſſotetras* obtaineth the roome of its *ſubſervient*, to the end, that by its Reſolver *Torp—Mu—Lag ☞ Myr*, being inſtructed how by cutting off the Logarithm of the *Radius*, from the ſumme of the Logarithms of the M. of one of the *firſt ſubtendents*, and ſecant complement of one of the *cathetoppoſits*, or Angles at the Baſe, reſiduats the Logarithm of the Tangent complement of the *Baſe* required, we may deliveredly extract, out of the generality of that propoſition, the peculiar Subordinate reſolutions of theſe three Problemets of *Vbamen*, viz. *Vſopet*, *Vdrbad*, and *Vthopheth*, varied (as you ſee) according to the diverſity of the *Præno ſcendas*, which being (as you were told already) the *firſt Baſal*, or great Baſe, the *firſt Baſet* or little Baſe, and the *firſt Co-baſe*; I will not detain you any longer upon this matter, but the rather haſten my tranſition to the other work, that in the Præno ſcendall partition of *Ammanepreb*, there is enough thereof ſet downe in the Table of my *Triſſotetras*.

The *Præno ſcendas* of *Ammanepreb*, or the three ſeverall *firſt Baſes*, conforme to the various nature of the *Caſes* thereof, being by the foreſaid *Datas* happily obtained, muſt concurre with each its correſpondent *Cathetoppoſite* (diſcernable, in their ſeverall qualities,

qualities, by the Characteristicks of πεφ.) for finding out of the perpendicular, which is the perpetuall *quæsitum* of the second operation. The *subservient* of this work is *Ethaner*, by whose Resolver, *To—Tag—Se☞Tyr*, we come to the knowledge of *Ethaners* three *Subdataquæres*, viz. *Ætopca*, *Ædobca*, and *Æthophca*, whereby we may perceive, that the same *quæsitum*, to wit, the perpendicular is obtained by the *Datas* of the three severall both Bases, and *Cathet opposite* Angles.

This so often mentioned perpendicular being thus made known, must, for the performance of the last and third work, joyne with the nixt *Cathetopposite*, the prime *Cathetopposite*, and the second *Co-cathetopposite*, as the *Case* will beare it, the *Datas* being the same in every point here, that in the last operation of the foregoing Mood (as by the subservients, *Exoman* and *Epsoner*, is obvious to any judicious Reader) thereby to obtaine the maine *quæsitum*, which in the first *Case*, by abstracting the fourth proportionall from the first great Base, in the second by abstracting from the fourth proportionall, the first little Base, and in the third by adding the fourth proportionall to another segment of the Base, is findable by any, that will undergoe the labour of adding, and substracting. For the acomplishment of this last operation *Epsoner* is the *Subservient*, by whose Resolver *Tag—Tolb—Te☞Syr*, we are taught how to deale with its three Subproblems, *Cathobœd*, *Cathopœt*, and *Cathopsœth*.

These last two operations being thus to the full extended, it remaineth now to treat how they ought to be in one compacted, or rather, for brevitie of computation, we should compact them both in one, before we take the paines to extend them: yet, because practice requireth one method, & the order of Doctrine another, we will, that we may be the lesse troublesome to the Readers memory, goe on (by ejecting some, and reserving other proportional termes) in our usuall course of conflating two Resolvers together. These Resolvers are in this Mood, *To—Tag—Se☞Tyr*, and *Tag—To—Te☞Syr*, the first thereof, sounding, *As the Radius, to the Tangent of one of the Cathetopposite Angles, or Angles at the Base: so the Sine of one of the first Bases, to the Tangent of the perpendicular:* and the second, *As the Tangent of one of the other Cathetopposite Angles to the Radius: so the Tangent of the perpendicular, to the sine of the side required.*

Here

Here may the Reader be pleaſed to conſider, that in all the gloſſe upon the poſterior operations of my *Diſergeticks*, I have beene contented to ſet downe (as he may ſee in the laſt two propoſitions) the bare Theorems of the *Reſolvers*, conforme to the nature of their Analogy, without troubling my ſelfe, or any body elſe, with repeating, or reiterating the way, how the Logarithms of the middle, and initiall termes are to be handled, for the obtaining of a fourth Logarithm; all that can be deſired therein, being to the full expreſſed already in my ample comments upon the Orthogonoſphericall Problems; to the which the induſtrious Reader, in caſe of doubting, may (if he pleaſe) have recourſe, without any great loſſe of time, or labour: however, for his better encouragement, I give another hint thereof in the cloſure of this Treatiſe.

But to returne where we left, ſeeing out of theſe two Reſolvers, *To—Tag—Se☞Tyr*, and *Tag—To—Te☞Syr*, according to the rules of coaleſceney, mentioned in both the *Moods* of *Ahalebmane*, both the Perpendicular and Radius may be ejected without any danger of loſing our aime of the maine *quæſitum*, it is evident, that the proportion of the Remanent termes, is, *Ta—Tag—Se☞Syr*, which comprehendeth both the laſt two *Reſolvers*, and the three foreſaid *Problemets* thereto belonging, and being interpreted, *As the Tangent of one Cathetoppoſite Angle, to the Tangent of another Cathetoppoſite: ſo the ſine of one of the firſt Baſes, to the ſine of a ſide, which uſhers in the ſide required.*

This generall Reſolver, according to the three ſeverall *Caſes* of this *Mood*, is to be particulariſed into ſo many finall Reſolvers; the firſt whereof, for *Dadiſſepamſor*, is *Tob-Top-Sat☞Sædfr*, Æt* Dyr*, that is, *As the Tangent of the next oppoſite, to the Tangent of the prime oppoſite: ſo the Sine of the firſt great Baſe, to the Sine of the ſecond little Baſe; which ſubducted from the foreſaid firſt great Baſe, will for the remainder afford us that ſegment of the Baſe, which is the ſide in the firſt Caſe required.*

Then for the ſecond Caſe, *Dadiſſexamſor*, the finall Reſolver is *Top—Tob—Sad☞Sætfr* Æd* Dyr*, that is, *As the Tangent of the prime Cathetoppoſite to the Tangent of the next oppoſite: ſo the Sine of the firſt Baſet, or little Baſe, to the Sine of the ſecond and great*

*great Base*; from which if we abstract the foresaid first little *Base*, the difference or remainer will be that Segment of the *Base*, which is the side demanded.

Lastly, for the Case *Dasimin*, the finall Resolver is *Tops—Toph—Sæth* ✕ *Sæthj* * *Æth* * *Syr*, that is, *As the Tangent of the second co-opposite, to the Tangent of the first co-opposite: so the Sine of the first co-base, to the Sine of the second co-base*; the summe of which two co-bases is the totall Base or side in the third *Case* required.

The reason of all this is proved by the third Disergetick Axiome, which is *Siubprortab*, as is pointed at by the first syllable of its Directory *Ammena*, and manifested to us in all the Analogies of this *Mood*, every one whereof runneth upon Tangents of *Angles*, and Sines of *Segments*, both to the *Base* belonging: nor can any doubt, that heares the resolution of the *Cases* of *Ammanepreb*, but that the habitude, which all the termes thereof have to one another, proceedeth meerly from *the reciprocall proportion, which the Tangents of the opposite Angles have to the Basal-segments, and contrariwise*.

The third Loxogonosphericall Disergetick *Figure*, and first of the *Laterals* (that is, the first two termes of whose *Datas* are *sides*, what ere the *quæsitum* be) is *Ebenabrole*, which comprehendeth all those Problems, wherein two *sides* being given, and an *Angle betweene*, either a *cathetopposite Angle*, or the third *side* is demanded. This *Figure*, conforme to the two severall *Quæsitas*, hath two *Moods*, to wit, *Enerablo*, and *Ennerable*.

The first *Mood* hereof, *Enerablo*, containeth all those obliquangularie questions, wherein *two sides with the Angle comprehended within them, being proposed, another Angle is required*, which *Angle* is alwayes one of the *Cathetopposites* or *Angles* at the *Base*, that is, either the complement to a Semicircle of the next *Cathetopposite, the prime Cathetopposite, or the second Cocathetopposite*: to the knowledge of all which, that we may with facility attaine, let us consider the generall *Maxim* of the *Cathetothesis* of this *Mood*, which is *Cafregpigeq*, that is to say, that the Perpendicular in all the Cases of *Enerablo* falleth from that given *side*, which is opposite to the *Angle* required, upon the other given *side*, continued, if need be; and according to the variety of the

*Angle*

*Angle* at the *Baſe* which is the *Angle* ſought for, there be theſe three eſpeciall *Tenets* of the generall *Maxims* of this *Mood*, viz. *Dacramfor*, *Damracfor*, and *Daſimquain*.

*Dacramfor*, which is the *Tenet* of the firſt *Caſe*, ſheweth, that *if the propoſed Angle be ſharp, and the required flat, the Perpendicular muſt fall outwardly.*

*Damracfor*, the *Tenet* of the ſecond *Caſe*, ſignifieth, that if a blunt, or obtuſe *Angle* be given, and an acute or ſharp demanded, the demiſſion of the Perpendicular muſt (as in the laſt) be externall.

Laſtly, *Daſimquain*, the *Tenet* of the third *Caſe*, ſheweth, that if the given, and required *Angles* be of the ſame nature, the Perpendicular muſt fall inwardly.

Having thus unfolded all the intricacies in my *Triſſotetras* of the *Cathetothetick* partition of this *Mood*, I may, without breaking order, ſtep back, to explicate what is contained in the preceding partition, and for the accompliſhing of the firſt Orthogonoſphericall work of this *Mood*, conſider what its *Pranoſcendas* are, and by what *Datas* they are to be obtained: but, ſeeing both the *Pranoſcendas*, and the *Datas*, together with the *ſubſervient*, and its *Reſolver*, with all the three *Subdatoquares*; and in a word, the whole contents of the firſt partition of this *Mood* of *Enerablo*, is the ſame in all and every jot with the *Pranoſcendas*, *Datas*, *Subſervient*, *Reſolver*, and *Problemets*, contained in the firſt partition of the laſt *Mood Ammanepreb*; I will not need to tell you any more, then that (the *Triſſotetras* it ſelfe (though otherwiſe ſhort enough) ſhewing that *Ubamen* is the *ſubſervient* to the *Pranoſcendas*: *Torp—Mu—Lag* & *Myr*, its *Reſolver*: and *Utopet*, *Udobad*, and *Uthopheth*, the three *Subproblems* both of this and the next preceding *Mood*) you be pleaſed to have recourſe to the gloſſe upon the laſt *Mood*, where this matter is treated of at large; to the which, for avoyding of repetition, I doe heartily recommend you.

The firſt work being thus expedited, we are to find out the Perpendicular by the ſecond, but ſo as that my direction to the Reader for the performance thereof ſhall detaine me no longer here, then the time I am willing to beſtow, in telling him, that the whole progreſſe of this operation, as well as of the preceding,

is amply expreſſed in my comment on the laſt *Mood*, from which, what ere is written of the Subſervient, *Ethaner*, its Reſolver, *To—Tag—Se☞Tyr*, or the under-problems, *Ætopca*, *Ædobca*, and *Æthophca*, thereby reſolved, may conveniently be tranſplaced hither, and reſeated there againe, without any prejudice to either; *Ammanepreb* being the onely *Mood*, which with this of *Enerablo* hath a baſal and oppoſite *catheteuretick* identity.

The Perpendicular, by theſe meanes being found out, muſt be employed in the laſt work of this *Mood*, to concurre with the ſecond *Baſidion*, or little *Baſe*, the ſecond great *Baſe*, and the ſecond *Co-baſe*, for obtaining of ſuch *Cathetoppoſites* as are, or uſher the maine *quæſitas*, which in the firſt *Caſe* is the complement of the fourth proportionall (*viz.* the next *Cathetoppoſite*) to a Semicircle; in the ſecond *Caſe* the prime *Cathetoppoſite*, and in the third, the ſecond *Cocathetoppoſite*. For the perfecting of this operation, *Erelan* is the Subſervient, by whoſe Reſolver, *Sei—Teg—To☞Tir*, we are inſtructed how to unfold its peculiar Problemets, *œdcathob*, *œtcathop*, and *œthcathops*.

All the three operations being thus ſingly accompliſhed, according to our wonted manner, the laſt two muſt be inchaced into one, and therefore their Reſolvers, *To—Tag—Se☞Tyr*, and *Sei—Teg—To☞Tir*, muſt be untermed of ſome of their proportionals: the which, that we may performe the more judiciouſly, let us conſider what they ſignifie apart; the firſt importeth (as in the laſt *Mood* I told you) that, *As the Radius is to the Tangent of one of the oppoſite Angles: ſo the Sine of one of the firſt Baſes, to the Tangent of the Perpendicular*: the ſecond ſoundeth, *As the Sine of one of the ſecond Baſes, to the Tangent of the Perpendicular: ſo the Radius, to the Tangent of an Angle, which either uſhers, or is the Angle required.*

Hereby it is evident, how the Radius is a multiplyer in the one, and a divider in the other, and that the Perpendicular, which with the Radius is a multiplyer in the ſecond row, is in the power of the three firſt termes of the firſt row, whereof the Radius is one, by vertue of all which, we muſt proceed juſt ſo with theſe laſt two operations here, as we have already done with the two laſt of the Moods of *Alamebna*, *Allamebne*, and *Ammanepreb*, and ejecting the Radius and Perpendicular out of both, inſtead of

*To—*

*To—Tag—Se☞Tyr* and *Sei—Teg—To☞Tir*; ſet downe *Sei—Tag—Se☞Tir*, that is, *As the Sine of one of the ſecond Baſes to the Tangent of one of the Cathetoppoſites: ſo is the Sine of one of the firſt Baſes, to the Tangent of one of the other Cathetoppoſites*: which propoſition comprehendeth to the full the laſt two operations, and according to the three ſeverall *Caſes* of this *Mood* is to be individuated into ſo many finall Reſolvers.

The firſt thereof, for *Dacramfor*, is *Sœd—Top—Sat☞Teb* Kir*, that is, *As the Sine of the ſecond Baſidion, or little Baſe, is to the Tangent of the prime Cathetoppoſite: ſo the Sine of the firſt, and great Baſe, to the Tangent of the next Cathetoppoſite*, whoſe complement to a Semicircle is the Angle required.

The ſecond finall Reſolver, is for *Damracfor*, the *Tenet* of the ſecond *Caſe*, and is *Sœt—Tob—Sad☞Top* Ir*, that is to ſay, *As the Sine of the ſecond, and great Baſe, to the Tangent of the next Cathetoppoſite: ſo the Sine of the firſt Baſidion, to the Tangent of the prime oppoſite*, which is the Angle required.

The third and laſt finall Reſolver, is for the third *Caſe Daſimquain*, and is couched thus, *Sœth—Toph—Sath☞Tops* Ir*, that is, *As the Sine of the ſecond Co-baſe is to the Tangent of the firſt Cocathetoppoſite: ſo is the Sine of the firſt Co-baſe to the Tangent of the ſecond Co-cathetoppoſite*, which is the Angle required.

The fundamentall reaſon of all this, is from the third Diſergetick Axiome *Sinbprortab*, the ſecond Determinater of whoſe Directory, *Ammena*, ſheweth that the *Mood* of *Enerablo*, in all the finall Reſolvers thereof, oweth the truth of its Analogy to the *Maxim* of *Sinbprortab*; becauſe of the *reciprocall proportion that is amongſt its termes, to be found betwixt the Sines of the baſall ſegments and the Tangents of the Cathetoppoſite Angles.*

The ſecond *Mood* of *Ebenabrole* is *Ennerable*, which comprehendeth all thoſe Obliquangulary Problems, wherein two ſides being given, with an Angle intercepted therein, the third ſide is demanded, which ſide is alwayes one of the *ſecond Subtendents*, that is either the *ſecond Subtendentine*, the *ſecond Subtendentall*, or the *ſecond Co-ſubtendent*: to the notice of all which, that we may the more eaſily attaine, let us perpend the generall Maxim of the *Cathetotheſis* of this *Mood*, *Cafregpag*; the meaning whereof is, that in this *Mood*, whatever the *Caſe* be, *the Perpendicular may fall*

*from the extremity of either of the given ſides, but muſt fall from one of them, oppoſite to the Angle propoſed, and upon the other given ſide, continued, if need be.*

Here may the Reader be pleaſed to obſerve, that the clauſe of the Perpendiculars falling oppoſite to the propoſed Angle, though it be onely mentioned in this place, might have as well beene ſpoke of in any one of the reſt of the Cathetothetick comments; becauſe it is a generall tie incumbent on the demiſſion of Perpendiculars in all Loxogonoſpherical Diſergetick Figures, whether Amblygonian or Oxygonian, that *it fall alwayes oppoſite to a knowne Angle, and from the extremity of a knowne ſide.*

Of this generall Maxim, *Cafregpaq*, according to the variety of the ſecond Subtendent, which is the ſide required, there be theſe three eſpeciall Tenets, *Dacforamb*, *Damforac*, and *Dakinatam*.

*Dacforamb*, the *Tenet* of the firſt *Caſe*, giveth us to underſtand, that if the given Angle be acute, and that one onely of the other two be an obtuſe Angle, the Perpendicular falleth outwardly.

*Damforac*, the *Tenet* of the ſecond *Caſe*, ſignifieth, that if the given Angle be obtuſe, and the other two acute, that the demiſſion of the Perpendicular is externall, as in the firſt.

Thirdly, *Dakinatam*, the *Tenet* of the third *Caſe*, and *variator* of the firſt, ſheweth, that if the propoſed Angle be of the ſame affection with one of the other Angles of the Triangle, as in the firſt *Caſe*, the Perpendicular may fall inwardly.

The *Cathetology* of this *Mood* being thus expeded, the *Pranoſcendas* thereof come next in hand to be diſcuſſed, which are the firſt *Baſes*, whoſe ſubſervient is *Vhamen*, and its Reſolver, *Terp—Mu—Lag ☞ Myr*, upon which depend the three *Subdatoquaras* of *Vtopar*, *Vdobed*, and *Vthapheth*.

Thus much I beleeve is expreſſed in the very Table of my *Triſſotetras*; and though a large explication might be with reaſon expected in this place, of what is but ſummarily mentioned there, yet becauſe what concerneth this matter, hath beene already treated of in the laſt two *Moods* of *Enerable*, and *Anmanepreb*, the whole diſcourſe whereof may be as conveniently peruſed, as if it were couched here, I will not dull the Reader with tedious rehearſals of one and the ſame thing, but, letting paſſe the pro-

greſſe

gresse of this first work, with the manner of which (by my former instructions, I suppose him sufficiently well acquainted) will proceed to the *Catheteuretick* operation of this *Mood*, and perpend by what *Datas* the perpendicular is to be found out.

To this effect, the *Prænoscendas* of *Ennerable*, to wit, the *first Basal*, the *first Basidion*, and the *first co-Base*, being by the last work already obtained, must concurre with each its correspondent first *subtendent*, viz. the first *Subtendentall*, the first *Subtendentine*, and the first *co-Subtendent*, discernable in their severall natures, by the *figuratives* of τ.δ.θ. for the perfecting of this second operation. The subservient of this work, is *Uchener*, by whose Resolver *Neg—To—Nu*☞*Nyr*, the three subproblems *Utæta*, *Utædca*, and *Uthæthca*, are made manifest: by vertue whereof it is perceivable, how the same *quæsitum* is attained unto by the *Datas* of three severall, both first *Subtendents*, and first *Bases*.

The perpendicular being thus obtained, must assist some other terme in the third operation, for the finding out of the maine *quæsitum*; which *quæsitum*, though it be different from the finall one of the last *Mood*, yet is the knowledge of them both attained unto, by meanes of the same *Datas*; the perpendicular, and the three second *Bases*, being ingredients in both.

It being certaine then, that the *perpendicular* must concurre in the last work of this *Mood* with the second *Basidion*, the second *Basal*, and second *co-Base*, for obtaining the second *Subtendentine*, the second *Subtendentall*, and second *co-Subtendent*; *Enerul*, is made use of for their *subservient*, by whose Resolver, *To—Neg—Ne*☞*Nur*, we are taught how to deale with its subordinat Problems, *Cathœdwd*, *Cathœtwt*, and *Cathœthwth*.

All the three works being thus specified apart, according to our accustomed Method, we will declare what way the last two are to be joyned into one; for the better effectuating wherof, their Resolvers, *Neg—To—Nu—*☞*Nyr*, and *To—Neg—Ne*☞*Nur*, must be interpreted; the first being, *As the sine complement of a first Base to the Radius: so the sine complement of a first subtendent, to the sine complement of the perpendicular.* And the second, *As the Radius, to the sine complement of a second Base: so the sine complement of the perpendicular, to the sine complement of a second subtendent*, which is the side required.

Now

Now, ſeeing a multiplier muſt be daſhed againſt a divider, being both quantified alike, and that all unneceſſary peſtring of a work with ſuperfluous ingredients is to be avoided; we are to deale with the Radius, and perpendicular in this place, as formerly we have done in the Moods of *Alamebna*, *Allamebne*, *Ammanepreb*, and *Enerablo*, where we did eject them forth of both the orders of proportionalls; and when we have done the like here, inſtead of *Neg—To—Nu* ⋋ *Nyr*, and *To—Neg—Ne* ⋋ *Nur*, we may with the ſame efficacie ſay, *Neg—Nu—Ne* ⋋ *Nur*, that is, *As the ſine complement of one ſide, is to the ſine complement of a ſubtendent: ſo the ſine complement of another ſide, to the ſine complement of another ſubtendent*; or more determinatly, *As the ſine complement of a firſt Baſe, to the ſine complement of a firſt ſubtendent: ſo the ſine complement of a ſecond Baſe, to the ſine complement of a ſecond ſubtendent.*

This theorem comprehendeth to the full both the laſt operations, and according to the number of the *Caſes* of this *Mood*, is particularized into three finall Reſolvers, the firſt whereof for the firſt *Caſe*, *Dacforamb*, is *Nat—Nut—Nœd* ⋋ *Nwd*yr*, that is, *As the ſine complement of the firſt Baſal, or great Baſe, to the ſine complement of the firſt Subtendentall, or great ſubtendent: ſo the ſine complement of the ſecond Baſidion, or little Baſe, to the ſine complement of the ſecond ſubtendentine, or little ſubtendent*, which is the ſide required.

The ſecond finall Reſolver, is for *Damforac*, the ſecond *Caſe*, and is ſet downe thus, *Ned—Nud—Nœt* ⋋ *Nwt*yr*, that is, *As the ſine complement of the firſt Baſidion, to the ſine complement of the firſt ſubtendentine: ſo the ſine complement of the ſecond Baſal, to the ſine complement of the ſecond ſubtendentall*, which is the ſide in this *Caſe* required.

The third, and laſt finall Reſolver is for *Dakinatamb*, and is expreſſed thus, *Nath—Nuth—Nœth* ⋋ *Nwth*yr*, that is to ſay, *As the ſine complement of the firſt co-baſe, to the ſine complement of the firſt co-ſubtendent: ſo the ſine complement of the ſecond co-baſe, to the ſine complement of the ſecond co-ſubtendent*, which in the third *Caſe* is alwayes the *ſide* required.

The reaſon of all this is proved out of the fourth, and laſt diſergetick Axiom, *Niubprodneſver*, whoſe directer *Ennerra*, ſheweth by its Determinater, the ſyllable *Enn*, that the *Datoquere* of *Enerable*,

*nerable*, is bound for the veritie of its proportion, in all the finall Reſolvers thereof, to the *maxime* of *Niubprodneſver*, becauſe of the direct analogie that, amongſt its termes, is to be ſeen betwixt *the ſines complements of the ſegments of the Baſe, & the ſines complements of the ſides of the verticall Angles*; which in all this Treatiſe, both for plaineſſe, and brevity ſake, I have thought fit to call by the names of *firſt* and *ſecond Subtendents*.

The fourth and laſt Loxogonoſpherical Diſergetick *figure*, and ſecond of the *Laterall*s, is *Eherolabme*, which is of all thoſe obliquangularie problems, wherin two ſides being given, and an oppoſite *Angle*, the interjacent *Angle*, or one of the other *ſides* is demanded; and, conforme to its two ſeverall *quæſitas*, hath two *Moods*, viz. *Erelomab*, and *Errelome*.

The firſt *Mood* hereof *Erelomab* comprehendeth all thoſe Loxogonoſpherical Problems, wherein two *ſides* with an oppoſite *Angle* being propoſed, the *Angle* between is demanded, which *Angle* is ſtill one of the verticals, that is, the firſt *verticall*, the ſecond *verticall*, or the double *vertical*: to the notice of all which, that we may the more eaſily attain, we muſt conſider the general *Maxim* of the *Cathetotheſis* of this *Mood*, which is *Cafriq*, the very ſame in name with the generall *Cathetothetick Maxim* of *Amanepra*, and thus far agreeing with it, that the Perpendicular in both muſt fal from the *Angle* required, and upon the ſide oppoſite to that *Angle*, increaſed if need be: but in this point different, that in *Amanepra*, the Perpendiculars demiſſion is from the *Angle* required upon the oppoſite *ſide*, conterminat with the two propoſed *Angles*, and in *Erelomab*, it falleth from the required *Angle*, upon the oppoſite *ſide* conterminat with the two propoſed *ſides*: and, according to the variety of the fourth proportionall, which, in the Analogies to this *Mood* belonging, uſhers in the verticall required, there be thoſe three eſpeciall *Tenets* of the generall *Maxim* of this *Mood*, viz. *Dacracforaur*, *Damracforeur*, and *Dacrambatin*.

*Dacracforaur*, which is the *Tenet* of the firſt *Caſe*, ſheweth, that if the given and demanded *Angles* be acute, and the third an obtuſe *Angle*, the Perpendicular falleth outwardly upon the third *ſide*, and the required *Angle* is a firſt verticall.

*Dambracforeur*, the *Tenet* of the ſecond *Caſe*, importeth, that if the propoſed *Angle* be obtuſe, and an acute *Angle* required;

the

the third *Angle* being acute, the Perpendicular muſt likewiſe in this *Caſe* fall outwardly upon the third *ſide*, and the *Angle* demanded be a ſecond verticall.

*Dacrambatin*, the *Tenet* of the third *Caſe*, ſignifieth, that if the propoſed *Angle* be acute, and an obtuſe *Angle* required, the Perpendicular falleth inwardly, and the demanded *Angle* is a double verticall.

I had almoſt forgot to tell you, that *Sindiſora* is the *Pluſ-minus* of this *Mood*, whereby we are given to underſtand, *that the ſumme of the top Angles, if the Perpendicular fall within, and their difference, if it fall without, is the Angle required:* and, ſeeing it varieth neither in name, nor interpretation from the *menſurator* of *Amanepra* (the diverſity betwixt them being onely in this, that the verticals there are inveſted with *Sines*, and here with *Sine* complements) I muſt make bold to deſire the Reader to look back to that place, if he know not why it is that ſome *Moods* are *Pluſ-minuſed*, and not others; for there he will find that *Sindiſoration* is meerly proper to thoſe *Caſes*, in the Analogies whereof the fourth proportionall is not the maine *quæſitum* it ſelfe, but the illaticious terme that brings it in.

The *Prænoſcendas* of the *Mood*, or *Quæſitas* of the firſt operation, falling next in order to be treated of, it is fitting we perpend of what nature they be in this *Mood* of *Erelomab*, that if they be different from thoſe of other *Moods*, we may, according to our accuſtomed diligence, formerly obſerved in the like occaſions, appropriate, in this parcell of the comment to their explication, for the Readers inſtruction, the greater ſhare of diſcourſe, the leſſe that before in any part of this Tractat, they have beene mentioned: But if it be ſo farre otherwiſe, that for their coincidence with other proturgetick *Quæſitas*, there can no materiall document concerning them be delivered here, which hath not beene ſpoke of already in ſome one or other of our foregoing *Datoquæres*, it were but an unneceſſary waſting of both time and paper to make repetition of that, which in other places we have handled to the full; and therefore, ſeeing the *Prænoſcendas* of this *Mood*, to wit, the double *top Angle* or verticall, the firſt *top Anglet* or *verticalin*, and the firſt *Co-top-Angle*, or *co-verticall*, together with the *Datas*, whereby theſe are obtained, *viz.* for the *ſide*, the

firſt

firſt *ſubtendentall*, the firſt *ſubtendentine*, and the firſt *co-ſubtendent*, and for the *Angle*, the prime *Cathetoppoſite*, the next *Cathetoppoſite*, and the firſt *Co-cathetoppoſite*, and conſequently the ſubſervient *Upalam*, its Reſolver *To—Tag—Nu—Mir*, and their three peculiar Problemets, *Utopat*, *Udoband*, and *Uthophauth*, are all and every one of them the ſame in this *Mood* of *Erelomab*, that they were in the three preceding *Moods* of *Alamebna*, *Allamebne*, and *Amanepra* (for theſe are the foure *Moods*, which have an Angulary *prænoſcendall* identity) we will not need (I hope) to talk any more thereof in this place, ſeeing what hath beene already ſaid concerning that purpoſe, will undoubtedly ſatisfie the deſire of any induſtrious civill Reader.

The *prænoſcendas* of the *Mood*, or the verticall *Angle*, according to the nature of the *Caſe*, being by the foreſaid *Datas* thus obtained, muſt needs concurre with each its correſpondent firſt ſubtendent, determined by the *figuratives* of *r. ſ. θ.* for finding out of the Perpendicular, of which work, *Ubamen* being the *ſubſervient*, by whoſe *Reſolver Nag—Mu—Torp—Myr*, the ſub-problems of *Utatca*, *Udaudca*, and *Uthauthca*, are made known, if I utter any more of this purpoſe, I muſt intrench upon what I ſpoke before in the ſecond operation of *Allamebne*, it being the onely *Mood* which, with this of *Erelomab*, hath a verticall, and ſubtendentine *Catheteuretick identity*.

The ſecond operation being thus accompliſhed, the perpendicular, which is alwayes an ingredient in the third work, muſt joyne with one of the rere ſubtendents for obtaining of the illatitious terme of the maine *quæſitum*: or, more particularly, by the concurrence of the Perpendicular with the ſecond ſubtendentine, the ſecond ſubtendentall, and ſecond *Co-ſubtendent*, according to the variety of the *Caſe*, we are to find out three *verticals*, which, by abſtracting the firſt from another *verticall*, then by abſtracting another *verticall* from the ſecond, and laſtly by adding the third *verticall* to another, afford the *ſumme*, and *differences*, which are the required *verticals*.

All this being fully ſet downe in my comment upon the Reſolutory partition of *Amanepra*, in which *Mood* the maine *quæſitum* is the ſame as here (though otherwiſe endowed) I need not any longer inſiſt thereon.

For the performance of this work, *Ukelamb* is the *subservient*, by whose Resolver *Meg—To—Mu☞☜Nir*, we are taught how to unfold the peculiar *problemes* of *Wdeathand*, *Wtcathat*, and *Wthcatheuth*.

All the three works being in this manner perfected, according to our accustomed method, we will shew unto you what way the last two are to be compacted in one: for the better expediting whereof, their Resolvers *Nag—Mu—To☞☜Myr*, and *Meg—To—Mu☞☜Nir*, must be explained, the first being, *As the Sine complement of an Angle, to the Tangent complement of a subtendent: so the Radius, to the Tangent complement of the side required*: Or, more particularly, *As the Sine complement of a verticall, to the Tangent complement of a first subtendent: so the Radius, to the Tangent complement of the Perpendicular*: And the second *Resolver* being, *As the Tangent complement of a given side, to the Radius: so the Tangent complement of a subtendent, to the Sine complement of a required Angle*: Or, more particularly, *As the Tangent complement of the Perpendicular, to the Radius: so the Tangent complement of a first subtendent, to the Sine complement of a verticall*, which ushers the *quæsitum*.

Now, seeing it falleth forth, that the Perpendicular, which is the fourth terme in the first order of proportionals, becometh first in the second row; and that in such an exigent (as I proved already for illustration of the same point in the *Mood* of *Amanepra*) the multiplyers and dividers of the first row must interchange their roomes, and consequently make the *Radius* ejectable, without any prejudice or hindrance to the progresse of the Analogy; and a place being left for the Perpendicular in both the rowes, without taking the paines to find out its value, because it is but a subordinate *quæsitum* for obtaining of the maine, and lieth hid in the power of the three first proportionals, instead of *Nag—Mu—To☞☜Myr*, and *Meg—To—Mu☞☜Nir*, we may, with as much truth and energy, say, *Mu—Nag—Mu☞☜Nir*, that is, *As the Tangent complement of a subtendent, to the Sine complement of an Angle: so the Tangent complement of another subtendent, to the Sine complement of another Angle*: Or, more particularly, *As the Tangent complement of a first subtendent, to the Sine complement of a verticall: so the Tangent complement of a second subten-*

*dent*

*dent, to the Sine complement of a verticall illative to the queſitum.*

This propoſition to the full containeth all that is in both the laſt operations, and, according to the number of the *Caſes* of this *Mood*, is ſpecialized into ſo many *finall Reſolvers*; the firſt whereof, for the firſt *Caſe Dacracforaur*, is *Mutnat—Mwd☞Neudfr*At*Aut*ir*, that is, *As the Tangent complement of the firſt ſubtendentall, to the ſine complement of the double verticall: ſo the tangent complement of the ſecond Subtendentine, to the ſine complement of the ſecond verticalin*, which ſubtracted from the *double verticall*, leaves the *firſt verticall* for the Angle required.

The ſecond finall Reſolver, is for *Damracforeur*, the ſecond *Caſe*, and is expreſſed thus, *Mud—Naud—Mwt☞Natfr* Aud*Eut*ir*, that is, *As the tangent complement of the firſt Subtendentine, to the ſine complement of the firſt verticalin: ſo the tangent complement of the ſecond Subtendentall, to the ſine complement of the double verticall*; from which if you deduce the firſt *verticalin*, there will remaine the ſecond *verticall* for the *Angle* required.

The laſt *finall Reſolver* is for the third *Caſe*, *Dacrambatin*, and is couched thus, *Muth—Nauth—Mwth☞Neuth*jauth*ir*, that is, *As the tangent complement of the firſt co-ſubtendent, to the ſine complement of the firſt co-verticall: ſo the Tangent complement of the ſecond co-ſubtendent, to the ſine complement of the ſecond co-verticall*, which, joyned to the firſt *co-verticall*, affordeth the *Angle* required.

The proofe of the veritie of all theſe Analogies, is taken out of the ſecond Diſergetick Amblygonoſpherieall Axiome, *Naverprortes*, the ſecond *Determinater* of whoſe *Directorie* ſheweth, that this *Mood* is one of its dependents; and with reaſon, becauſe of the reciprocall Analogie, that amongſt its termes is perceivable betwixt the Tangents of the verticall ſides, which in this *Mood* are alwayes firſt ſubtendents, and the ſine-complements of the verticall Angles; that is to ſay (according to the literal meaning of my finall Reſolvers of this *Mood*) the *direct proportion that is betwixt the tangent-complements of the verticall ſides, or rere ſubtendents, & the ſine-complements of the vertical Angles*, for the proportion is the ſame with that, wherof I have told you ſomewhat already in the *Mood* of *Allamebme*, the fellow dependent of *Erelomab*.

The ſecond *Mood* of *Eberelabme*, fourth of the *Laterals*,

eighth of the *Sphericobliquangularie Diſergeticks*, twelfth of the *Loxogonoſphericalls*, eight and twentieth of the *Sphericals*, and one and fourtieth or laſt of the *Triangulars*, is *Errelome*, which comprehendeth all thoſe obliquangularie Problems, wherein two *ſides* being given with an oppoſite *Angle*, the third *ſide* is required, which *ſide* is alwayes either one of the *ſegments* of the *Baſe*, or the *Baſe* it ſelfe: to the knowledge of all which, that we may reach with eaſe, we muſt perpend the generall *Maxim* of the *Cathetotheſis* of this *Mood*, which is *Cacurgyq*, that is to ſay, *the Perpendiculars demiſſion, in all the Caſes of Errelome, muſt be from the concurſe of the given ſides, upon the ſide required, continued, if need be.*

The *Pluſ-minus* of this *Mood* is *Sindiforiu*, which importeth, that if the Perpendicular fall internally, the *ſumme* of the ſegments of the Baſe, or the totall Baſe, is the ſide demanded: and if it fall without, the difference of the Baſes (the little Baſe, being alwayes but a ſegment of the greater) is the maine *quæſitum.*

The *Mood* of *Ammanepreb* is *ſindiforated* in the ſame manner as this is; becauſe the maine *Quæſitas*, and fourth proportionals of both doe in nothing differ, but that thoſe are ſinuſed, and theſe run upon ſine-complements.

The proſilience of the Perpendicular in all ſpherical Diſergeticks, being ſo neceſſary to be knowne (as I have often told you) becauſe of the facility thereby to reduce them to Rectangulary operations, it falleth out moſt conveniently here, according to the method propoſed to my ſelfe, to ſpeak ſomewhat of the three ſeverall *Tenets* of the *Cathetotheſis* of this *Mood*, and what is underſtood by *Dakyxamſor*, *Dambyxamſor*, and *Dakypambin.*

*Dakyxamſor*, which is the Tenet of the firſt *Caſe*, declareth, that if the propoſed *Angle* be acute, and the *ſide* required conterminate with an obtuſe *Angle*, the demiſſion of the Perpendicular is extrinſecall.

*Dambyxamſor*, the Tenet of the ſecond Caſe, importeth, that if the given *Angle* be obtuſe, and that the ſide required be annexed thereto, the Perpendicular muſt, as in the laſt, fall outwardly.

Thirdly, *Dakypambin*, the Tenet of the laſt *Caſe*, ſignifieth, that if the *angle* propoſed be ſharp, & that the demanded ſide be ſubjacent

to

to an obtuſe or blunt *Angle*, the Perpendicular falleth inwardly.

Having thus proceeded in the enumeration of the Cathetothetick Tenets of this *Mood*, according to the manner by me obſerved in thoſe of all the former Diſergeticks, ſave the firſt, I am confident the Reader (if he hath peruſed all the *Tractat* untill this place) will not think ſtrange why, *Dakypambin* being but the third, I ſhould call it the *Tenet* of the laſt *Caſe* of this *Mood*; for though in *Alamebna* I ſpoke ſomewhat of every Amblygonoſpheric all Diſergetick *Moods* generall Cathetothetick maximes diviſion into foure eſpeciall *Tenets*, appropriable to ſo many ſeverall *Caſes*: yet the fourth *Caſe*, viz. that wherein all the *Angles* are homogeneall, whether blunt or ſharp, not being limited to any one *Mood*, but adæquatly extended to all the eight, it ſeemed to me more expedient to let its generality be known by mentioning it once or twice, then (by doing no more in effect) to make ſuperfluous repetitions; and, as in the firſt Diſergetick *Caſe*, for the Readers inſtruction, I did under the name of *Simomatin*, explicate the nature thereof: ſo, for his better remembrance, have I chooſed rather to ſhut up my Cathetothetick comment with the ſame diſcourſe wherewith I did begin it, then unneceſſarily to weary him with frequent reiterations, and a tedious rehearſall of one and the ſame thing in all the ſix ſeverall intermediat *Moods*.

It is not amiſſe now, that the perpendicularity of this *Mood* is diſcuſſed, to conſider what the *prænoſcendas* thereof are, or the *Quæſitas* of the firſt operation: but, as I ſaid in the laſt *Mood*, that there is no need to inſiſt ſo long upon the explication of thoſe *prænoſcendas*, whereof ample relation hath beene already made in ſome of my Proturgetick comments, as upon thoſe others, which, for being altogether different from ſuch as have beene formerly mentioned, claim (by the law of parity, in their imparity) right to a large diſcourſe apart, I will confine my pen upon this ſubject, within thoſe preſcribed bounds, and ſeeing the *firſt Baſal*, the *firſt Baſidion*, and *firſt Co-baſe*, together with the *Datas*, whereby they are found out, *viz.* for the *ſide*, the *firſt ſubtendentall*, the *firſt ſubtendentine*, and the *firſt co ſubtendent*; and for the *Angle* the prime *Cathetoppoſite*, the *next Cathetoppoſite*, and the *firſt Co-cathetoppoſite* (the *Datas* being both for *ſide* and *Angle* the

the same here, that they were in the former *Mood*) then the Subservient *Ubamen*, and its Resolver *Torp—Mu—Lag* ☞ *Myr*, with the three peculiar *Problemets* thereto belonging, *Utopat*, *Udobad*, and *Vthophath*, are all and every one of them the same in this *Mood* of *Errelome*, that they were in the three foregoing *Moods* of *Ammanepreb*, *Enerablo*, and *Ennerable*, these being the onely foure *Moods* which have a laterall prænoscendal identity, the Reader will not (in my opinion) be so prodigall of his owne labour, nor covetous of mine, that either he would put himselfe, or me to any further paines, then have beene already bestowed upon this matter by my selfe for his instruction; and therefore, leaving it for a supposed certainty, that the *Prænoscendas*, or first *Bases* (according to the nature of the *Case*) cannot escape the Readers knowledge, by what hath beene by me delivered of them; I purpose here to give him notice, that these foresaid *first Bases* must concurre with each its correspondent *first Subtendent*, to wit, the *first subtendentall*, the *first subtendentine*, and *first co-subtendent*, dignoscible by the *Characteristicks* of τ. δ. θ. for obtaining of the Perpendicular, of which operation, *Vchener* being the *Subservient*, by whose Resolver *Neg—To—Nu* ☞ *Nyr*, the *Problemets* of *Utatca*, *Udadca*, and *Uthathca*, are made manifest, as to the same effect it remaines couched in my comment upon *Ennerable*, which is the onely *Mood*, that, with this of *Errelome*, hath a *subtendentine* and *Basal Catheteuretick* identity.

The second work being thus perfected, the perpendicular, thereby found out, is to assist one of the rere subtendents, in obtaining the illatitious terme of the maine *quasitum*, correspondent thereto, discernable by the *Characteristicks* or *Figuratives* of δ. τ. θ. or, more plainly to expresse it, the Perpendicular must concurre (according as the *Case* requires it) with the *second subtendentine*, the *second subtendentall*, and *second co-subtendent*, (as you may see in the last *Mood*, the *Datas* of the Resolutory partition whereof are the same as here) to find out three *Bases*, which, by abstracting the first from another *Base*, then by abstracting another *Base* from the second; and lastly, by adding the third *Base* to another, afford the *summe* and *differences*, which are the required *Bases*.

For the performance of this operation, the same Subservient and

and Reſolver ſuffice, which ſerved for the laſt: ſo that *Uchener ſubſerveth* it, by whoſe Reſolver *Neg—To—Nu☞Nyr*, we are inſtructed how to explicate the *Subdatoquæres* of *Wdcathœd*, *Wtcathœt*, and *Wthcathœth*, or more orderly *Cathwdœd*, *Cathwtœt*, and *Cathwthœth*.

All the three works being thus accompliſhed, the manner of conflating the laſt two in one reſts to be treated of; for the better perfecting of which deſigne, the two Reſolvers, or the ſame in its greateſt generality doubled, *viz. Neg—To—Nu☞Nyr*, and *Neg—To—Nu☞Nyr*, muſt be interpreted: The truth is, both of them, as they ſound in their vaſteſt extent of ſignification, expreſſe the ſame Analogy, without any difference, which is, *As the Sine complement of a given ſide, to the Radius: ſo the Sine complement of a ſubtendent, to the Sine complement of another ſide:* but when more contractedly, according to the ſpecification of the *ſide*, they doe ſuppone ſeverally, they ſhould be thus expounded; the firſt, *As the Sine complement of a firſt Baſe, to the totall Sine: ſo the Sine complement of a firſt ſubtendent to the Sine complement of the Perpendicular*: and the ſecond, *As the Sine complement of the Perpendicular, to the totall Sine: ſo the Sine complement of a ſecond ſubtendent, to the Sine complement of a ſecond Baſe*, which uſhers the main *quæſitum*.

Now, the *Perpendicular*, and *Radius*, being both to be expelled theſe two foreſaid orders of proportionall termes, for the reaſons which, in the laſt preceding *Mood*, and ſome others before it, I have already mentioned, and which to repeat (further then that the ſympathy of this place with that may be manifeſted in the tranſ-ſeating of multiplyers and dividers, occaſioned by the fourth terme in the firſt rowes, being firſt in the ſecond) is altogether unneceſſary: in lieu of *Neg—To—Nu☞Nyr*, and *Neg—To—Nu☞Nyr*, we may ſay, with as much truth, power, and efficacie, and farre more compendiouſly, *Nu—Ne—Nu☞Nyr*, that is, *As the Sine complement of a ſubtendent, to the Sine complement of a ſide: ſo the ſine complement of another Subtendent, to the Sine complement of another ſide:* Or, more particularly, and appliably to the preſent Analogy, *As the Sine complement of a firſt ſubtendent, to the Sine complement of a firſt Baſe: ſo the Sine complement of a ſecond ſubtendent, to the Sine comple-*

*ment*

*ment of a ſecond Baſe*, illative to the *quæſitum*.

This *theorem*, or *propoſition*, comprehendeth in every point all that is in the two laſt operations, and, not tranſcending the number of the *Caſes* of this *Mood*, is divided into ſo many finall Reſolvers; the firſt whereof for the firſt *Caſe*, *Dakyxamſor* is, *Nut—Nat—Nwd* ☞ *Nœdſr* **Æt** *Dyr*, that is, *As the Sine complement of the firſt ſubtendentall, to the Sine complement of the firſt Baſall: ſo the Sine complement of the ſecond ſubtendentine, to the Sine complement of the ſecond Baſe*; which ſubducted from the firſt *Baſal*, reſiduats the *ſegment* that is the *ſide* required.

The ſecond finall Reſolver of this *Mood*, and that which is for the ſecond *Caſe* thereof, *Dambyxamſor*, is *Nud—Nad—Nwt* ☞ *Nœtfr* **Æd** *Dyr*, that is, *As the Sine complement of the firſt ſubtendentine, to the Sine complement of the firſt Baſidion: ſo the Sine complement of the ſecond ſubtendentall, to the Sine complement of the ſecond Baſall*; which, the firſt *Baſidion* being ſubtracted from it, leaves, for Remainder, or difference, that *ſegment* of the *Baſe*, which is the ſide demanded.

The laſt finall Reſolver of this *Mood* (belonging to the third *Caſe*, *Dakypambin*, as alſo to the fourth, *Simomatin*, (if what we have already ſpoke of that matter will permit us to call it the fourth) for *Simomatin*, together with the third *Caſe* of every *Mood*, is ſtill reſolved by the laſt finall Reſolver thereof) is *Nuth—Nath—Nwth* ☞ *Nœthj* **Æth** *Syr*, that is, *As the Sine complement of the firſt co-ſubtendent, to the Sine complement of the firſt Co-baſe: ſo the Sine complement, of the ſecond Co-ſubtendent, or alterne ſubtendent, to the Sine complement of the ſecond Co-baſe or alterne Baſe*; which added to the firſt *Co-baſe*, ſummes an Aggregat of ſubjacent *ſides*, which is the totall *Baſe*, or *ſide* required.

The fundamentall ground of the truth of theſe Analogies, is in the fourth and laſt Amblygonoſphericall Axiome, *Ninbprodneſver*; (as we are made to underſtand by the ſecond *determinater* of its Directory *Ennerra*) for by the direct proportion that, amongſt the terms thereof, is viſible, (*viz.* betwixt the Sines complements of the ſubtendents, or Sides of the verticall Angles, and the ſegments of the Baſes, and inverſedly) it is apparent, that this *Mood* doth no leſſe firmely depend upon it, then that of *Ennerable* formerly explained.

Now

Now, with reaſon doe I conjecture, that, without diſappointing the Reader of his expectation, I may here ſecurely make an end of this Trigonometricall Treatiſe; becauſe of that *Triſſotetrall* Table, which comprehendeth all the Myſteries, Axiomes, Principles, Analogies, and Precepts of the Science of Triangular Calculations, I have omitted no materiall point unexplained: yet ſeeing, for avoyding of prolixity, I was pleaſed in my comment upon the eighth Loxogonoſpherical Diſergeticks, barely to expreſſe in their finall Reſolvers, the Analogie of the termes, without putting my ſelfe to the paines I took in my Sphericorectangulars, how to order the Logarithms, and Antilogarithms of the proportionalls, for obtaining of the maine *Quæſitas*, and that by having to the full explicated the variety of the proportions of the foreſaid *Moods*, and upon what ſeverall *Axiomes* they doe depend, thereby making the way more pervious, thorough Logarithmicall difficulties, for the Readers underſtanding, I deliberatly propoſed to my ſelfe this method at firſt, and choſe, rather then diſperſedly to treat of thoſe things in the gloſſe (where, by reaſon of the diſturbed order, the correſpondencie or reference to one another of theſe Sphericobliquangulary *Datoquares*, could not by any meanes have beene ſo conceivable) to ſummon their appearance to the Cataſtrophe of this Tractat, that, having them all in a front before us, we may the more eaſily judge of the ſemblance, or diſſimilitude of their proportionalities, and what affinity, or relation, whether of parity or imparity is amongſt their reſpective proportionall terms: all which, both for intelligibility and memory, are quicklier apprehended, and longer retained, by being accumulatively reſerved to this place, then if they had beene each in its proper cell (though never ſo amply) diſcourſed upon apart.

Here therfore, that the Reader may take a generall view at once of all the Diſergetick Amblygonoſphericall analogiſed ingredients, ready for Logarithmication, I have thought fit to ſet downe a Liſt of all the eight forenamed *Moods*, together with the Finall Reſolvers, in their ampleſt extent thereto belonging, in the manner as followeth.

| | | | | | |
|---|---|---|---|---|---|
| Alamebna. | *Say-Nag-Sa* ☞ *Nir* | } | Enerablo. | *Sei-Tag-Se* ☞ *Tir* | |
| Allamebne. | *Nag-Mu-Na* ☞ *Mur* | | Ennerable. | *Neg-Nu-Ne* ☞ *Nur* | |
| Amanepra. | *Na-Say-Nag* ☞ *Sir* | | Erelomab. | *Mu-Nag-Mu* ☞ *Nir* | |
| Ammanepreb. | *Ta-Tag-Se* ☞ *Syr.* | | Errelome. | *Nu-Ne-Nu* ☞ *Nyr.* | |

Theſe being the eight Diſergeticks, attended by their Adæquat finall Reſolvers, it is not amiſſe, that we examine them all one after another, and ſhew the Reader how, with the help of a convenient Logarithmicall Canon, he may eaſily out of the Analogie of the three firſt termes of each of them, frame a computation apt for the finding out of a fourth proportionall, to every ſeverall ternarie correſpondent: and ſo in order, beginning at the firſt, we will deale with *Say—Nag—Sa* ☞ *Nir* (which is the Adæquat finall Reſolver of *Alamebna*, and compoſed (as it is appropriated to the firſt *Mood* of the Diſergeticks) of the *Sines* of *verticals*, and the *Anti-ſines* of *Cathetoppoſites*) and ſo proceed therein, that by adding to the ſumme of the Sine of a verticall, and Co-ſine of a Cathetoppoſite, the Arithmetical complement of the Sine of another verticall, we will be ſure (cutting off the ſupernumerary digit or digits towards the left) to obtaine the Co-ſine of the Cathetoppoſite required, which *Cathetoppoſites* and verticals are particulariſed according to the *Caſes* of the *Mood*.

The ſecond is, *Nag—Mu—Na* ☞ *Mur*, which, running upon the *Anti-ſines* of *verticals*, and the *Co-tangents* of *ſubtendent ſides*, ſheweth, that if to the Aggregat of a firſt hypotenuſall Co-tangent, and verticall Anti-ſine, we joyne the Arithmeticall complement of the Anti-ſine of another verticall, (obſerving the uſuall preſection) we cannot miſſe of the Co-tangent of the ſecond ſubtendent ſide required, which both *ſecond*, and *firſt ſubtendents* have their peculiar denominations, according to the *Caſes* of the *Mood*.

The third Reſolver is, *Nu--Say--Nag* ☞ *Sir*, which, being nothing elſe but the firſt inverted, runneth the ſame very way upon the Anti-ſines of Cathetoppoſites, and ſines of verticals: and therefore doth the unradiuſed ſumme of the Anti-ſine of a Cathetoppoſite, the ſine of a verticall, and the Arithmeticall complement of the Anti-ſine of another Cathetoppoſite, afford the ſine of the verticall, illatitious to the Angle required; which

*verticals*

*verticals* and *Cathetoppoſites* are particulariſed according to the variety of the *Caſes* of this *Sindiforating Mood.*

The fourth generall Reſolver is *Ta—Tag—Se☞Syr*, which, courſing on the Tangents of all the Cathetoppoſites, and ſines of all the Baſes, evidenceth, that the ſumme of the Tangent of a Cathetoppoſite, and ſine of a firſt Baſe, added to the Arithmeticall complement of the Tangent of another Cathetoppoſite (unradiated) is the ſine of the ſecond Baſe, illative to the ſegment required; which *Baſes* (both *firſt* and *ſecond*) and *Cathetoppoſites*, are ſpecialiſed conform to the *Caſes* of this *Sindiforiuting Mood.*

The fifth Reſolver is, *Sei—Tag—Se☞Tir*, which, compoſed of the ſines of the ſecond and firſt Baſes, and the Tangents of Cathetoppoſites, giveth us to know, that if to the ſumme of the ſine of a firſt Baſe, and the Tangent of a verticall, we adde the Arithmeticall complement of the ſine of a ſecond Baſe, (not omitting the uſuall preſection) we cannot faile of the Tangent of the Cathetoppoſite required, which *Cathetoppoſites*, and *Baſes*, both *firſt* and *ſecond*, are particulariſed according to the *Caſes* of the *Mood.*

The ſixth generall Reſolver is, *Neg—Nu—Ne☞Nur*, which, running along the Co-ſines of all the Baſes and Subtendents, ſheweth, that by the ſumme of the Co-ſines of a ſecond Baſe, and firſt ſubtendent joyned with the Arithmeticall complement of the Co-ſine of a firſt Baſe (if we obſerve the cuſtomary preſection) we find the ſecond Subtendent required, which both *firſt* and *ſecond Subtendents*, together with the *firſt* and *ſecond Baſes*, are all of them particulariſed conforme to the *Caſes* of the *Mood.*

The ſeventh Reſolver is, *Mu—Nag—Mu☞Nir*, which, courſing along the Anti-tangents of firſt, and ſecond Subtendents, and the Anti-ſines of verticals, ſheweth, that the ſumme of the Anti-tangent of a ſecond Subtendent, and Anti-ſine of a verticall, together with the Arithmeticall complement of the Anti-tangent of a firſt Subtendent (the uſuall preſection being obſerved) is the Anti-tangent of that verticall, which uſhers in the verticall required; all which, both *Verticals*, and *Subtendents*, both *firſt*, and *ſecond*, have their peculiar denominations conforme to the *Caſes* of this *Sindiforating Mood.*

The eighth and laſt generall Reſolver is, *Nu—Ne—Nu—Nyr*, which (running altogether upon Co-ſines of Subtendents, and Baſes, both firſt, and ſecond of either, and is nothing elſe but the ſixth inverted) ſheweth, that the ſumme of the Co-ſines of a ſecond ſubtendent, and firſt Baſe, with the Arithmeticall complement of the co-ſine of a firſt Subtendent, (obſerving the uſuall preſection) affords the Co-ſine of the ſecond Baſe, illatitious to the ſegment required; which *Baſes* and *Subtendents*, both *firſt*, and *ſecond*, are peculiarly denominated according to the ſeverall *Caſes* of this *Sindiforiuting Mood*.

Thus have I finiſhed the *Logarithmication* of the generall Reſolvers of the Loxogonoſphericall Diſergeticks, ſo farre as is requiſite, wherein I often times mentioned the Arithmeticall complement of Sines, Co-ſines, Tangents, and Co-tangents: and though I ſpoke of that purpoſe ſufficiently in my Spherico-rectangular comments, yet, for the Readers better remembrance thereof, I will once more define them here.

The Arithmeticall complements of Sines are Co-ſecants; of Co-ſines, Secants; of Tangents, Co-tangents; and of Co-tangents, Tangents; each being the others complement to the double Radius: but if ſuch a Canon were framed, wherein the ſingle Radius is left out of all Secants, and Tangents of major Arches, then would each be the others complement to the ſingle Radius, and all Logarithmicall operations in queſtions of Trigonometry ſo eaſily performable by addition onely, that ſeldome would the preſectionall digit exceed an unit.

Having already ſaid ſo much of theſe eight Diſergeticks, I will conclude my diſcourſe of them with a ſummary delineation of the eight ſeverall Concordances which I obſerved amongſt them; for either they reſemble one another in the *Datas* of their *Moods*, or in their *Proturgetick* operations, or in their dependance upon the ſame *Axiome*, or in the work of *perpendicular* finding, or in their *Datas* for the main demand, or in their materiall *Quæſitas* (though diverſly endowed) or in their *inverſion*, or laſtly in their *ſindiforation*, which affinity is onely betwixt two paires of them, as the firſt two amongſt two quaternaries apeece, and the next five between foure couples each one, the brief hypotypoſis of all which is here expoſed to the view of the Reader.

CONCOR-

# CONCORDANCES.

## Datall.

Datangulary.
1. *Alamebna.* 3. *Amanepra.*
2. *Allamebne.* 4. *Ammanepreb.*

Datolaterall.
1. *Enerablo.* 3. *Erelomab.*
2. *Ennerable.* 4. *Errelome.*

## Prænoſcendall.

Verticall.
1. *Alamebna.* 3. *Amanepra.*
2. *Allamebne.* 4. *Erelomab.*

Baſall.
1. *Ammanepreb.* 3. *Ennerable.*
2. *Enerablo.* 4. *Errelome.*

## Theorematick.

Nabadproſver|Naverprortes.
1. *Alamebna.* 1. *Allamebne.*
2. *Amanepra.* 2. *Erelomab.*

Siubprortab|Niubprodneſver.
1. *Ammanepreb.* 1. *Ennerable.*
2. *Enerablo.* 2. *Errelome.*

## Catheteuretick.

Oppoverticall|Hypoverticall.
1. *Alamebna.* 1. *Allamebne.*
2. *Amanepra.* 2. *Erelomab.*

Oppobaſall|Hypobaſall.
1. *Ammanepreb.* 1. *Ennerable.*
2. *Enerablo.* 2. *Errelome.*

## Datyſterurgetick.

Cathetoverticall|Oppocathetall.
1. *Alamebna.* 1. *Amanepra.*
2. *Allamebne.* 2. *Ammanepreb.*

Cathetobaſall|Hypocathetal.
1. *Enerablo.* 1. *Erelomab*
2. *Ennerable.* 2. *Errelome.*

## Zetetick.

Cathetoppoſite|Hypotenuſall.
1. *Alamebna*S.* 1. *Allamebne* M.*
2. *Enerablo*T.* 2. *Ennerable*N.*

Verticall|Baſall.
1. *Amanepra*S.* 1. *ammanepreb.*S*
2. *Erelomab*N.* 2. *Errelome* N.*

## Inverſionall.

Sinocoſinall|Sinocotangentall.
1. *Alamebna.* 1. *Allamebne.*
2. *Amanepra.* 2. *Erelomab.*

Tangentoſinall|Coſinocoſinall.
1. *Ammanepreb.* 1. *Ennerable.*
2. *Enerablo.* 2. *Errelome.*

## Sindiforall.

Sindiforatall.
1. *Amanepra.* 2. *Erelomab.*

Sindiforiutall.
1. *Ammanepreb.* 2. *Errelome.*

The

# THE EPILOGUE.

WHat concerneth the reſolving of all manner of Triangles, whether *plain*, or *Spherical*, *Rectangular*, or *Obliquangular*, being now (conform to my promiſe in the Title) to the ful explained, commented on, perfected, and with all poſſible brevity, and perſpicuity, in all its abſtruſeſt and moſt difficult Secrets, from the very firſt principles of the Science it ſelfe, made manifeſt, proved, and convincingly demonſtrated; I will here ſhut up my diſcourſe, and bring this Tractat to a period: which I may do with the more alacrity, in that I am confident, there is no Precept belonging to that faculty which is not herein included, or reducible thereto: and therefore (I beleeve) the judicious Reader will not be fruſtrate of his expectation, though by cutting the threed of my Gloſſe, I doe not illuſtrate what I have written with variety of examples; ſeeing practically to treat of Triangulary calculations, in applying their doctrine to uſe, were to digreſſe from the purpoſe in hand, and incroach upon the ſubject of other Sciences; a priviledge, which I muſt decline, as repugnant to the ſcope propoſed to my ſelfe, in keeping this book within the ſpeculative bounds of *Trigonometry*: for, as *Logica utens*, is the Science to the which it is applyed, and not *Logick*: So doth not the matter of *Trigonometry*, exceed the Theory of a *Triangle*: And as *Arithmeticall*, *Geometricall*, *Aſtronomicall*, *Phyſicall*, and *Metaphyſicall* definitions, diviſions, and argumentations, are no part of the Art that inſtructeth how to define, divide, and argue, nor matter incumbent to him that teacheth it: even

ſo,

ſo, by divulging this Treatiſe, doe I preſent the Reader with a Key, by meanes whereof he may enter into the chiefeſt treaſures of the *Mathematicall* Sciences; for the which, in ſome meaſure, I deſerve thanks, although I help him not to unſhut the Coffers wherein they lie incloſed: for, if the Lord chamberlain of the Kings houſhold ſhould give me a Key, made to open all the doores of the Court, I could not but graciouſly accept of it, though he did not goe along with me to try how it might fit every lock. The application is ſo palpable, that, not minding to inſiſt therein, I will here ſtop the current of my Pen, and by a circulary concluſion, ending where I begun, certifie the Reader, that if he intend to approve himſelfe an Artiſt in matters of *Pleuſiotechnie*, *Poliechyrologie*, *Coſmography*, *Geography*, *Aſtronomy*, *Geodeſie*, *Gnomonicks*, *Sciography*, *Catoptricks*, *Dioptricks*, and many other moſt exquiſite Arts and Sciences, Practical and Theoretick, his ſureſt courſe, for attaining to ſo much knowledge, is to be well verſed in *Trigonometry*, to underſtand this Treatiſe aright, revolve all the paſſages thereof, ruminate on the *Table*, and peruſe the *Triſſotetras*.

# A *Lexicidion* of ſome of the hardeſt words, that occurre in the diſcourſe of this inſtitution *Trigonometricall.*

BEing certainly perſwaded, that a great many good ſpirits ply *Trigonometry*, that are not verſed in the learned Tongues, I thought fit, for their encouragement, to ſubjoyne here the explication of the moſt important of thoſe Greek, and Latin termes, which, for the more efficacy of expreſſion, I have made uſe of in this Treatiſe: in doing whereof, that I might both inſtruct the Reader, and not weary him, I have endeavoured perſpicuity with ſhortneſſe: though (I ſpeak it ingenuouſly) to have been more prolixe therin, could have coſt but very little labor to me, who have already bin pretty well verſed in the like, as may appear by my Etymologicall dictionary of above twenty ſeven thouſand proper names, mentioned in the Lemmas of my ſeverall Volums of Epigrams, the words whereof are for the moſt part abſtruſer, derived from moe Languages, and more liable to large, and ample interpretations. However (*cæteris paribus*) brevity is to be preferred; therefore let us proceed to the Vocabulary in hand.

THE

# THE LEXICIDION.

## A.

ACute, *comes from* Acuo, acuere, *to ſharpen, and is ſaid of an Angle, whoſe including ſides, the more that its meaſure is leſſe then a Quadrant, have their concurſive, and angulary point the more penetrative, ſharp, keen and piercing: Whence an acutangled triangle.*

Adæquat, *is that, which comprehendeth to the full, whatever is in the thing to the which it is compared, and for the moſt part in my* Triſſotetras *is ſaid of the generall finall* Reſolvers, *in relation to the* Moods *reſolved by them. It is compounded of* Ad, *and* æquo, æquare, parem facere, *to make one thing altogether like, or equall to another.*

Adjacent, *ſignifieth to lie neare, and cloſe, and is applyed both to* ſides, *and* Angles, *in which ſenſe likewiſe I make uſe of the words adjoyning the, conterminat, or conterminall with, annexed to, intercepted in, and other ſuch like, for the more variety, as adherent, bounding, bordering, and ſo forth: It comes from* Adjaceo, Adjacere, *to lie neere unto, as the words* Ad *and* jaceo, *which are the parts whereof it is compounded, moſt perſpicuouſly declare.*

Additionall, *is ſaid of the Line, which, in my* comment, *is indifferently called the Line of Addition, the Line of continuation, the extrinſecall Line, the exceſſe of the* Secant *above the* Radius, *the Reſiduum, or the new Secant: it comes from* Addo, Addere, *which is compounded of* Ad, *and* do, *to put to and augment.*

Affection, *is the nature, paſſion, and quality of an Angle, and conſiſteth either in the* obtuſity, acuteneſſe, *or* rectitude *thereof: It is a verball from* Afficio, affeci, affectum, *compounded of* ad, *and* facio.

Aggregat, *is the ſumme, totall, or reſult of an Addition, and is compounded of* Ad, *and* grex; *for, as the Shepheard gathers his Sheep into a flock, ſo doth the Arithmetician compact his numbers to be added into a ſumme.*

Alternat, *is ſaid of Angles, made by a Line cutting two or more parallels, which Angles may be properly called ſo; becauſe they differ in nothing elſe but their ſituation; for if the ſectionary Line, to the which I ſuppoſe the parallels to be fixed, have the higheſt and loweſt points thereof to interchange their ſites, by a motion progreſſive towards the roome of the under* Alternat, *and terminating in that of the upper one, we will find, that both the inclination of the Lines towards one another, and the quality of the Angles, will, notwithſtanding that alteration, be the ſame as before; hence it is that they are called* alternat, *becauſe there is no other difference betwixt them: or, if* alternat *be taken (as arithmetically it is) for that proportion, wherein the Antecedent is compared to the Antecedent, and the Conſequent to the Conſequent, the ſenſe will likewiſe hold in the foreſaid Angles; for if by the paralleliſme of two right Lines, cut with a third, two blunt, and two keen Angles be produced (as muſt needs, unleſſe the Secant line be to the parallels a perpendicular) the keen or acute Angle will be to its complement, or ſucceſsively following obtuſe Angle, as the other acute unto its following obtuſe; therefore alternly, as the Antecedents are to one another,* viz. *the Acute to the Acute: ſo the Conſequents, the obtuſe to the obtuſe. And if the Angles be right, the direct, and alternat proportion is one and the ſame; the third, and fourth terms of the Analogy being in nothing different from the firſt, and ſecond.*

Ambient, *is taken for any of the legs of a rectangle, or the including, containing, or comprehending ſides of the right Angle: it comes from* Ambio, Ambire, *which is compounded of* Am *and* eo, *i. e.* circumeo: *and more properly applied to both, then to any one of them, though uſually it be uſurped for one alone,* vide Leg.

Amblygonian, *is ſaid of obtuſe angled Triangles, and Amblygonoſpherical of obtuſe ſphericals: It is compoſed of* Ἀμβλὺς, *and* γωνία. angulus.

Amfractuoſities, *are taken here for the cranklings, windings, turnings, and involutions belonging to the equiſoleary Scheme; of* am *and* frango, quod ſit quaſi via crebris mæandris undequaque interrupta.

Analogy,

Analogy, *ſignifieth an equality of proportion, a likeneſſe of reaſons, a conveniencie, or habitude betwixt termes: It is compounded of* ἀνὰ, æqualiter, *and* λόγος, ratio.

Analytick, *reſolutory, and is ſaid of thoſe things that are reſolved into their firſt principles, of* ἀνὰ, re, *and* λύω, ſolvo.

Antilogarithm, *is the Logarithm of the complement; as for example, the Anti-logarithm of a Sine is the Logarithm of the Sine complement,* vide Logarithm.

Anti-ſecant, *Anti-ſine, and Anti-tangent, are the complements of the Secant, Sine, and Tangent, and are called ſometime Co-ſecant, Co-ſine, and Co-tangent: they have* anti *prefixed, becauſe they are not in the ſame columne, and* co, *becauſe they are in the next to it.*

Apodictick, *is that, which is demonſtrative, and giveth evident proofs of the truth of a concluſion; of* ἀπὸ, *and* δείκνυμι, monſtro, oſtendo, unde ἀπόδειξις, demonſtratio.

Area, *is the capacity of a Figure, and whole content thereof.*

Arch, *or* Ark, *is the ſegment of a circumference leſſe then a ſemicircle: major Arch is above* 45. *degrees, a minor Arch, leſſe then* 45. videCircle.

Arithmeticall *complement, is the difference betweene the Logarithm to be ſubſtracted, and that of the double, or ſingle Radius.*

Artificiall *numbers, are the Logarithms, and artificiall Sine the Logarithm of the Sine.*

Axiome, *is a maxim, tenet, or neceſſary principle, whereupon the Science of a thing is grounded: it cometh from* ἄξιος, dignus; *becauſe ſuch things are worthy our knowledge.*

## B.

BAſall, *adjectively is that, which belongeth to the Baſe, or the ſubjacent ſide, but ſubſtantively the great Baſe.*

Baſangulary, *is ſaid of the Angles at the Baſe.*

Baſidion, *or baſet, is the little Baſe, all which come from the Greek word* Cαίνω, βάσις.

Baſiradius, *is the totall Sine of that Arch, a Segment whereof is the Baſe of the propoſed ſphericall Triangle.*

Biſected, *and Biſegment, are ſaid of lines cut into two equall parts: it comes from* biſeco, biſecare, biſectum, biſegmen.

Bluntneſſe, *or flatneſſe, is the obtuſe affection of Angles.*

Bucarnon, *by this name is entitled the ſeven and fortieth propoſition of the firſt of the elements of* Euclid; *becauſe of the oxe, or, (as ſome ſay) the hecatomb, which* Pythagoras, *for gladneſſe of the invention, ſacrificed unto the gods: of* βοῦς, bos, *and* ἀγνυμι, viciſſim aliquid capio; *they being (as it is ſuppoſed) well pleaſed with that acknowledgement of his thankfulneſſe for ſo great a favour, as that was, which he received from them: you may ſee the propoſition in the ſeventeenth of my Apodicticks.*

C.

CAnon, *is taken here for the Table of Sines and Tangents, or of their Logarithms: it properly ſignifieth the needle or tongue of a balance, and metaphorically a rule, whereby things are examined.*

Caſes, *are the parts wherein a Mood is divided from* cado.

Cathetos, *is a Perpendicular line, from* καθίημι, demitto, *of* κατὰ, *and* ἵημι.

Catheteuretick, *is concerning the finding out of the Perpendicular of* Κάθετος, *and* ἑυρίσκω, ἑυρήσω, invenio.

Cathetobaſall, *is ſaid of the Concordances of Loxogonoſpherical Moods, in the Datas of the Perpendicular, and the Baſe, for finding out of the maine quæſitum.*

Cathetoppoſite, *is the Angle oppoſite to the Perpendicular; it is a hybrid or mungrell word, compoſed of the Greek* κάθετος, *and Latin* oppoſitus.

Cathetorabdos, *or Cathetoradius, is the totall Sine of that Arch, a Segment whereof is the Cathetos, or Perpendicular of the propoſed Orthogonoſpherical.*

Cathetotheſis, *and cathetothetick are ſaid of the determinat poſition of the Perpendicular, which is ſometimes expreſſed by cathetology, inſtructing us how it ſhould be demitted: of* κάθετος, *and* θέσις, *from* τίθημι, pono, colloco.

Cathetoverticall, *is ſaid of the Concordances of Loxogonoſpherical Moods in the Datas of the perpendicular, and the verticall Angle in the laſt operation.*

Catoptrick, *the Science of perſpective, from* κατόπτομαι, perſpicio, cerno.

Characteriſtick, *is ſaid of the letters, which are the notes and marks of diſtinction, called ſometimes ſignratives, or determiniters, from* χαράσσω, ſculpo, imprimo.

Circles,

Circles, *great circles are thoſe which biſect the Sphere, leſſer Circles thoſe which not.*

Circular *parts, are in oppoſition to the reall and naturall parts of a Triangle.*

Circumjacent, *things which lie about, of* circum *and* jaceo.

Coaleſcencie, *a growing together, a compacting of two things in one; it is ſaid of the laſt two operations of the Loxogonoſphericals conflated into one, from* coaleſco *or* coaleo, *of* con *and* alo.

Cobaſe, *a fellow Baſe, or that which with another Baſe hath a common Perpendicular, of* con *and* baſis.

Cocathetoppoſite, *is ſaid of two, Angles at the Baſe, oppoſite to one and the ſame Cathetos.*

Coincidence, *a falling together upon the ſame thing, from* coincido, *of* con, *and* incido, *ex* in, *&* cado.

Comment, *is an interpretation, or expoſition of a thing, and comes from* comminiſcor, comminiſci, mentionem facere.

Compacted, *joyned, and knit together, put in one; from* compingo, compegi, compactum, *vide* Coaleſcencie.

Complement, *ſignifieth the perfecting that which a thing wanteth, and uſually is that, which an Angle or a Side wanteth of a Quadrant, or* 90. *degrees: and of a Semicircle, or* 180. *from* compleo, complere, *to fill up.*

Concurſe, *is the meeting of lines, or of the ſides of a Triangle, from* concurro, concurſum.

Conflated, *compacted, joyned together, from* conflo, conflatum, conflare, *to blow together,* vide Inchaſed.

Conſectary, *is taken here for a Corollary, or rather a ſecondary Axiome, which dependeth on a prime one, & being deduced from it, doth neceſſarily follow. From* conſector, conſectaris, *the frequentative of* conſequor.

Conſound, *to ſound with another thing; it is ſaid of conſonants, which have no vocality without the help of the vowell.*

Conſtitutive, *is ſaid of thoſe things, which help to frame, make, and build up: From* conſtituo, *of* con *and* ſtatuo.

Conſtitutive *ſides, the ingredient ſides of a Triangle.*

Conſtructive *parts, are thoſe, whereof a thing is built, and framed: From* conſtruo, conſtructum, *to heap together, and build up, of* con *and* ſtrues.

Conterminall, *is that which bordereth with, and joyneth to a thing, of*

con

con *and* terminus, vide Adjacent, *or* Inſident.

Cordes, *and cordall, are ſaid of ſubtenſes metaphorically; becauſe the Arches and ſubtenſes are as the bow and ſtring:* chorda, *comes of the Greek word* χορδὴ, inteſtinum, ilia, quia ex illis chordæ conficiuntur.

Correctangle, *that is one which, with another rectangle, hath a common Perpendicular.*

Correſpondent, *that which anſwereth with, and hath a reference to another thing, of* con *and* reſpondeo.

Coſinocoſinall, *is ſaid of the Concordances of Loxogonoſpherical Moods, agreeing, in that the termes of their finall Reſolvers run upon Coſines.*

Coſmography, *is taken here for the Science whereby is deſcribed the celeſtiall Globe, of* κόσμος, *and* γράφω.

Coſubtendent, *is the ſubtendent of a correctangle, or that which with another is ſubſterned to two right Angles, made by the demiſſion of one and the ſame Perpendicular.*

Coverticall, *is the fellow top Angle, from whence the Perpendicular falleth.*

## D.

DAta, *is ſaid of the parts of a Triangle, which are given us, whether they be ſides or Angles, or both, of* do, datum, dare.

Datimiſta, *are thoſe Datas, which are neither Angles onely, nor ſides onely, but Angles, and ſides intermixedly: of* data, *and* miſta, *from* miſceo.

Datangulary, *is ſaid of the Concordances of thoſe Moods, for the obtaining of whoſe Præneſcendas, we have no other Datas, but Angles, unto the foreſaid Moods common.*

Datapurall, *comes from* datapura, *which be thoſe Datas, that are either meerly Angles, or meerly ſides.*

Datolaterall, *is ſaid of the Concordances of thoſe Moods, for the obtaining of whoſe Præneſcendas, the ſame ſides ſerve for Datas.*

Datoquære, *is the very Problem it ſelfe, wherein two or three things are given, and a third or fourth required, as by the compoſition of the word appears.*

Datiſterurgetick, *is ſaid of thoſe Moods which agree in the Datas of the laſt work: of* data, ὕστερον, poſtremum, *and* ἔργον, opus.

Demiſſion,

Demiſſion, *is a letting fall of the Perpendicular : from* demitto, demiſſum.

Determinater, *is the characteriſtick or figurative letter of a directory: from* determinare, *to preſcribe and limit.*

Diagonall, *taken ſubſtantively, or* diagonie, *is a line drawn from one Angle to another, of* διὰ *and* γωνία· *what the* diagonie *is in ſurfaces, the* axle *is in ſolids.*

Diagrammatiſe, *to make a Scheme or Diagram, from* διαγράφω, delineo.

Diatypoſis, *is a briefe ſummary deſcription, and delineation of a thing : or the couching of a great deale of matter, for the inſtruction of the Reader, in very little bounds, and in a moſt neat and convenient order : from* διατυπόω, inſtituo, item melius diſpono, vide Ὑποτύπωσις.

Diodot, *is* Pythagoraſes Bucarnon, *or the gift beſtowed on him by the gods : of* Διὸς, *the genitive of* Ζεὺς, *and* δοτὸς, datus, *from* δίδωμι, do, *vide* Bucarnon.

Dioptrick, *the art of taking heights and diſtances, from* διόπτομαι pervidendo, altitudinem dimenſionemque turrium & murorum exploro.

Directly, *is ſaid of two rowes of proportionals, where the firſt terme of the firſt row, is to the firſt of the ſecond, as the laſt of the firſt, is to the laſt of the ſecond.*

Directory, *is that which pointeth out the Moods dependent on an Axiome.*

Diſcrepant, *different, diſſonant, id eſt,* diverſo modo crepare.

Diſergeticks, *of two operations, of* δὶς, *and* ἔργον.

Document, *inſtruction, from* doceo.

## E.

ELucidation, *a clearing, explaining, reſolving of a doubt, and commenting on ſome obſcure paſſage, from* elucido, elucidare.

Energie, *efficacie, power, force; from* ἐνεργὸς, qui in opere eſt, *of* ἐν *and* ἔργον, opus.

Enodandum, *that which is to be reſolved and explicated, declared, and made manifeſt, from* enodo, enodare, *to unknit, or cut away the knot.*

Equation,

Equation, *or rather* aquation, *a making equall, from* æquo, æquare.

Equiangularity, *is that affection of Triangles, whereby their Angles are equall.*

Equicrurall, *is ſaid of Triangles, whoſe legs or ſhanks are equall;* of æquale, *and* crus, cruris; *leg being taken here for the thigh and leg.*

Equilaterall, *is ſaid of Triangles, which have all their ſides, ſhanks, or legs equall, of* æquale *and* latus, lateris.

Equipollencie, *is a ſamenеſſe, or at leaſt an equality of efficacie, power, vertue, and energie;* of æquus *and* polleo.

Equiſolea, *and* Equiſolearie, *are ſaid of the grand Orthogono ſphericall Scheme; becauſe of the reſemblance it hath with a horſe-ſhoe, and may in that ſenſe be to this purpoſe applied with the ſame metaphoricall congruencie, whereby it is ſaid, that the royall army at Edge-hill was imbatteld in a half-moon.*

Equivalent, *of as much worth and vertue, of* æquus *and* valeo.

Erected, *is ſaid of Perpendiculars, which are ſet or raiſed upright upon a Baſe, from* erigere, *to raiſe up, or ſet aloft.*

Externall, *extrinſecall, exteriour, outward, or outer, are ſaid of teſt of Angles, which being without the Area of a Triangle, are comprehended by two of its ſhanks meeting or cutting one another, accordingly as one or both of them are protracted beyond the extent of the figure.*

## F.

FAciendas, *are the things which are to be done:* faciendum *is the gerund of* facio.

Figurative, *is the ſame thing as Characteriſtick, and is applied to thoſe letters which doe figure and point us out a reſemblance and diſtinction in the Moods.*

Figures, *are taken here for thoſe partitions of Trigonometry, which are divided into Moods.*

Flat, *is ſaid of obtuſe, or blunt Angles.*

Forwardly, *is ſaid of Analogies, progreſſive from the firſt terme to the laſt.*

Fundamentall, *is ſaid of reaſons, taken from the firſt grounds and principles of a Science.*

G.

G.

GEodeſie, *the Art of Surveying, of* γῆ, *or* γαῖα, terra, *and* δαίω, divido, partior.

Geography, *the Science of the Terreſtriall Globe, of* γῆ terra, *and* γράφω, deſcribo.

Gloſſe, *ſignifieth a Commentary, or explication, it cometh from* γλῶσσα.

Gnomon, *is a Figure leſſe then the totall ſquare, by the ſquare of a Segment: or, according to* Ramus, *a Figure compoſed of the two ſupplements, and one of the Diagonall ſquares of a Quadrat.*

Gnomonick, *the Art of Dyalling, from* γνώμων, *the cock of a Dyall.*

Great Circles, *vide* Circles.

H.

HOmogeneall, *and* Homogeneity, *are ſaid of Angles of the ſame kind, nature, quality, or affection: from* ὁμογένεια, communio generis.

Homologall, *is ſaid of ſides congruall, correſpondent, and agreeable,* viz. *ſuch as have the ſame reaſon or proportion from* ὁμολογία, ſimilis ratio.

Hypobaſall, *is ſaid of the Concordances of thoſe Loxogonoſphericall Moods, which, when the Perpendicular is demitted, have for the Datas of their ſecond operation the ſame Subtendent and Baſe.*

Hypocathetall, *is ſaid of thoſe which for the Datas of their third operation have the ſame Subtendent and Perpendicular.*

Hypotenuſall, *is ſaid of Subtendent ſides, from* ὑπὸ *and* τείνω.

Hypotypoſis, *a laying downe of ſeverall things before our eyes at one time, from* ὑποτυπόω, oculis ſubjicio, delineo, & repræſento, *vide* Διατύπωσις.

Hypoverticall, *is ſaid of Moods, agreeing in the ſame Catheteuretick Datas of ſubtendent and verticall, as the Analyſis of the word doth ſhew.*

I

IDentity, *a ſameneſſe, from* idem, *the ſame.*

Illatitious, *or illative, is ſaid of the terme which bringeth in the* quæſitum, *from* infero, illatum.

Inchaſed, *coagulated, fixed in, compacted, or conflated, is ſaid of the laſt two Loxogonoſpherical operations put into one,* vide Compacted, Conflated, *and* Coaleſcencie.

Including ſides, *are the containing ſides of an Angle of what affection ſoever it be,* vide, Ambients, Legs, *&c.*

Individuated, *brought to the loweſt diviſion,* vide, Specialiſed, *and* Specification.

Indowed, *is ſaid of the termes of an Analogie, whether ſides, or Angles, as they ſtand affected with Sines, Tangents, Secants, or their complements,* vide Inveſted.

Ingredient, *is that which entreth into the compoſition of a Triangle, or the progreſſe of an operation, from* ingredior, *of* in *and* gradior.

Initiall, *that which belongeth to the beginning, from* initium, ab ineo, ſignificante incipio.

Inſident, *is ſaid of Angles, from* inſideo, *vide* Adjacent, *or* Conterminall.

Interjacent, *lying betwixt, of* inter *and* jaceo; *it is ſaid of the Side or Angle betweene.*

Intermediat, *is ſaid of the middle termes of a proportion.*

Inverſionall, *is ſaid of the Concordances of thoſe Moods which agree in the manner of their inverſion; that is in placing the ſecond and fourth termes of the Analogy, together with their indowments, in the roomes of the firſt and third, and contrariwiſe.*

Inveſted, *is the ſame as indowed, from* inveſtio, inveſtire.

Irrationall, *are thoſe which are commonly called ſurd numbers, and are inexplicable by any number whatſoever, whether whole, or broken.*

Iſoſceles, *is the Greek word of* equicrurall, *of* ἴσος, ἴσον, *and* σκέλος, crus.

## L.

Laterall, *belonging to the ſides of a Triangle, from* latus, lateris.

Leg, *is one of the including ſides of an Angle, two ſides of every Triangle being called the Legs, and the third, the Baſe; the Legs therefore or ſhankes of an Angle are the bounds inſiſting or ſtanding upon the Baſe of the Angle.*

Line *of interception, is the difference betwixt the Secant, and the Radius, and is commonly called the* reſiduum.

Logarithms, *are thoſe artificiall numbers, by which, with addition and ſubtraction,*

*ſubtraction onely, we work the ſame effects, as by other numbers, with multiplication and diviſion: of* λόγος, ratio, proportio, *and* ἀριθμος, numerus.

Logarithmication, *is the working of an Analogy by Logarithms, without having regard to the old laborious way of the naturall Sines, and Tangents; we ſay likewiſe Logarithmicall and Logarithmically, for Logarithmeticall, and Logarithmetically; for by the ſyncopiſing of* et, *the pronunciation of thoſe words is made to the eare more pleaſant: a priviledge warranted by all the dialects of the Greek, and other the moſt refined Languages in the world.*

Loxogonoſpherical, *is ſaid of oblique ſpherical, of* λόξος, obliquus, *and* σφαιρικὸς, ad ſphæram pertinens, *from* σφαῖρα, globus.

## M.

MAjor *and* Minor *Arches*, vide Arch.

Maxim, *an axiome, or principle, called ſo (from* maximus) *becauſe it is of greateſt account in an Art or Science, and the principall thing we ought to know.*

Meane, *or middle proportion, is that, the ſquare whereof is equall to the plane of the extremes: and called ſo becauſe of its ſituation in the Analogy.*

Menſurator, *is that, whereby the illatitious terme is compared, or meaſured with the maine* quæſitum.

Monotropall, *is ſaid of figures, which have one onely Mood, of* μόνος, *and* τρόπος, *from* τρέπω.

Monurgeticks, *are ſaid of thoſe Moods, the maine* Quæſitas *whereof are obtained by one operation, of* μόνος, *and* ἔργον.

Moods, *determine unto us the ſeverall manners of Triangles, from* modus, *a way, or manner.*

## N.

NAturall, *the naturall parts of a Triangle, are thoſe of which it is compounded, and the circular, thoſe whereby the maine* quæſitum *is found out.*

Neareſt, *or next, is ſaid of that Cathetoppoſite Angle, which is immediatly oppoſite to the perpendicular.*

Notandum, *is ſet downe for an admonition to the Reader, of ſome remarkable thing to follow, and is the Gerund of* Noto, notare.

## O.

OBlique, *and obliquangulary, are ſaid of all Angles that are not right.*

Oblong, *is a parallelogram, or ſquare more long then large: from* oblongus, *very long.*

Obtuſe, *and obtuſe angled are ſaid of flat, and blunt Angles.*

Occurſe, *is a meeting together, from* occurro, occurſum.

Oppobaſall, *is ſaid of thoſe Moods, which have a Catheteuretick Concordance in their Datas of the ſame Cathetoppoſite Angles, and the ſame Baſes.*

Oppocathetall, *is ſaid of thoſe Loxogonoſphericals which have a Datiſterurgetick Concordance in their Datas of the ſame Angles at the Baſe, and the Perpendicular.*

Oppoverticall, *is ſaid of thoſe Moods which have a Catheteuretick Concordance in their Datas of the ſame Cathetoppoſites, and verticall Angles.*

Orthogonoſpherical, *is ſaid of right angled Sphericals, of* ὀρθὸς, rectus, γωνία, angulus, *and* σφαῖρα, globus.

Oxygonoſpherical, *is ſaid of acute-angled ſpherics, of* ὀξὺς, γωνία, *and* σφαῖρα.

## P.

PAralleliſme, *is a Parallel, equality of right lines, cut with a right line, or of Sphericals with a Spherical, from* παραλλήλος, equidiſtans *of* παρὰ *and* ἄλληλος.

Parallelogram, *is an oblong, long ſquare, rectangle, or figure made of parallel lines: of* παραλλήλος, *and* γράμμη, linea.

Partiall, *is ſaid of* enodandas *depending on ſeverall Axioms.*

Particulariſe, *ſpecialiſe, by ſome eſpeciall difference to contract the generality of a thing.*

Partition, *is ſaid of the ſeverall operations of every Loxogonoſpherical Mood, and is divided in prænoſcendall, catheteuretick and hyſterurgetick.*

Permutat *proportion, or proportion by permutation, or alternat proportion,*

*on, is when the Antecedent is compared to the Antecedent, and the Conſequent to the Conſequent*, vide Perturbat.

Perpendicularity, *is the affection of the Perpendicular, or plumb-line; which comes from* perpendendo, id eſt, explorando altitudinem.

Perturbat, *is the ſame as* permutat; *and called ſo becauſe the order of the Analogie is perturbed.*

Planobiquangular, *is ſaid of plaine Triangles, wherein there is no right Angle at all.*

Planorectangular, *is ſaid of plaine right-angled Triangles.*

Planotriangular, *is ſaid of plaine Triangles, that is, ſuch as are not Sphericall.*

Pleuſeotechnie, *the Art of Navigation, of* πλεύσις, πλεύσιας, navigatio, *and* τέχνη, ars.

Pluſminuſed, *is ſaid of Moods which admit of Menſurators or whoſe illatitious termes are never the ſame, but either more or leſſe then the maine* quæſitas.

Poliechyrologie, *the Art of fortifying Townes and Cities, of* πόλις, urbs, civitas, ἐχυρόω, munio firmo, *and* λόγος, ratio.

Poſſubſervient, *is that which after another ſerveth for the reſolving of a queſtion; of* poſt, *and* ſubſerviens: *of* ſub *and* ſervio.

Potentia, *is that wherein the force and whole reſult of another thing lies.*

Power, *is the ſquare, quadrat, or product of a line extended upon it ſelfe, or of a number in it ſelfe multiplied.*

Powered, *ſquared, quadrified.*

Precept, *document, from* præcipio, præceptum.

Prænoſcenda, *are the termes, which muſt be knowne before we can attaine to the knowledge of the maine quæſitas; of* præ *and* noſco.

Prænoſcendall, *is ſaid of the Concordances of thoſe Moods, which agree in the ſame prænoſcendas.*

Præſection, *præſectionall, is concerning the digit towards the left, whoſe cutting off ſaveth the labour of ſubtracting the double or ſingle Radius.*

Præſcinded problems, *are thoſe ſpeculative Datoquares, which are not applied to any matter by way of practice.*

Præſubſervient, *is ſaid of thoſe Moods which in the firſt place we muſt make uſe of for the explanation of others; of* præ, *and* ſubſervio.

Prime, *is ſaid of the furtheſt Cathetopoſite, or Angle at the Baſe, contained within the Triangle to be reſolved.*

Primifie,

Primifie the Radius, *is to put the* Radius *in the first place*, primumque inter terminos collocare proportionales.

Problem, problemet, *a queſtion or datoquare, from* προβάλλομαι, unde πρόβλημα propoſitum, objectaculum.

Product, *is the reſult*, factus, *or* operatum *of a multiplication, from* produco, productum.

Proportion, *proportionality, are the ſame as Analogy, and Analogiſme; the firſt being a likeneſſe of termes, the other of proportions.*

Propoſition, *a propoſed ſentence, whether theorem, or problem.*

Proſiliencie, *is a demiſſion, or falling of the Perpendicular, from* proſilio, ex pro & ſalio.

Proturgetick, *is ſaid of the firſt operation of every Diſergetick Mood, of* πρῶτον *and* ἔργον, *the* ον *being Attically contracted into* ω.

## Q.

Quadrant, *the fourth part of a Circle.*

Quadranting, *the protracting of a Spherical ſide unto a Quadrant.*

Quadrat, *a Square*, a forma quadræ, *the power or poſſibility of a line*, vide, Power.

Quadrobiquadræquation, *concerneth the Square of the ſubtendent ſide, which is equall to the Biquadrat, or two Squares of the Ambients.*

Quadroſubduction, *is concerning the ſubtracting of the Square of one of the Ambients, from the Square of the Subtendent.*

Quæſitas, *the things demanded, from* quæro, quæſitum.

Quotient, *is the reſult of a diviſion, from* quoties, *how many times.*

## R.

Radically meeting, *is ſaid of thoſe Oblongs, or Squares, whoſe ſides doe meet together.*

Radius, *ray, or beame, is the Semidiameter, called ſo metaphorically, from the ſpoake of a wheele which; is to the limb thereof, as the Semidiameter, to the circumference of a circle.*

Reciprocall, *is ſaid of proportionalities, or two rowes of proportionals, wherein the firſt of the firſt is to the firſt of the ſecond, as the laſt of the*

*the ſecond is to the laſt of the firſt, and contrarily.*

Rectangular, *is ſaid of thoſe figures, which have right Angles.*

Reſinedly, *is ſaid when we go the ſhorteſt way to work by primifying the Radius.*

Renvoy, *a remitting from one place to another, it comes from the French word* Renvoyer.

Repreſentative, *is ſaid of the letters, which ſtand for whole words; as* E. *for ſide,* L. *for ſecant,* U. *for ſubtendent.*

Reſiduat, *is to leave a remainder,* nempe id quod reſidet & ſupereſt.

Reſolver, *is that which looſeth and untieth the knot of a difficulty, of* re *and* ſolvo.

Reſolutory, *is ſaid of the laſt partition of the Loxogonoſpherical operations.*

Reſult, *is the laſt effect of a work.*

Root, *is the ſide of a Square, Cube, or any coſſick figure.*

## S.

SCheme, *ſignifieth here the delineation of a Geometricall figure, with all parts neceſſary for the illuſtrating of a demonſtration, from* ἔχω habeo.

Sciography, *the Art of ſhadowing, of* σκία, umbra, *and* γράφω, ſcribo.

Segment, *the portion of a thing cut off,* quaſi ſecamentum, quod a re aliqua ſecatur.

Sexageſimat, *ſubſexageſimat, reſubſexageſimat, and bireſubſexageſimat, are ſaid of the diviſion, ſubdiviſion, reſubdiviſion, and reer-reſubdiviſion of degrees into minuts, ſeconds, thirds, and fourths, in* 60. *of each other: the deviſor of the fore-goer being ſucceſtively the following dividend, and the quotient alwayes ſixty.*

Sharp, *is ſaid of acute Angles.*

Sindiforall, *is ſaid of thoſe Moods, the fourth terme of whoſe Analogie is onely illatitious to the maine quæſitum.*

Sindiforation, *is the affection of thoſe foreſaid Moods, whereby the value of the menſurator is knowne.*

Sindiforatall, *is concerning thoſe Moods, whoſe illatitious terme is an Angle.*

Sindiforiutall, *is of thoſe Moods, whoſe illatitious terme is a ſide; all theſe foure words are compoſed of repreſentatives, and (if I remember well) mentioned in my explanation.*

Sino-

Sinocoſinall, *is ſaid of the Concordances of thoſe Moods, which agree in this, that their Analogies run upon ſines, and ſine-complements.*

Sinocotangentall, *is ſaid of thoſe Moods, which agree in that the termes of their Analogie run upon Sines and Tangent-complements.*

Sinus, *is ſo called (I beleeve) becauſe it is alwayes in the very boſome of the Circle.*

Sinuſed, *is ſaid of termes endowed or inveſted with Sines.*

Specialized, *contracted to more particular termes,* vide, Individuated.

Specifying, *determinating, particulariſing.*

Specification, *a making more eſpeciall, by contracting the generality of a thing,* vide, Specialized.

Sphericodiſergeticks, *are the Spherical Triangles of two operations.*

Structure *of an operation, is the whole frame thereof, from* ſtruo, ſtructum.

Subdatoquære, *is a particular datoquære, and is applied to the problems of the caſes of every Sphericodiſergetick Mood,* vide, Sub-problems.

Subjacent, *is the ſubſterned ſide or the Baſe, of* ſub & jaceo, *vide,* Suſtentative, Suſtaining ſide *and* Subſterned.

Subordinate *problems, is the ſame with ſubdatoquære.*

Subproblems, *is the ſame with ſubordinate problems, or problemets.*

Subſervient, *is ſaid of Moods which ſerve in the operation of other Moods.*

Subſterned, *is the ſubjacent ſide or Baſe: or, more generally, any ſide oppoſite to an Angle; of* ſub *and* ſterno, ſternere, *vide,* Subjacent.

Subtendent, *is the ſide oppoſite to the right Angle, of* ſub *and* tendo; *as if you would ſay, Under-ſtretched.*

Subtendentine, *is the ſubtendent of a little rectangled Triangle, comprehended within the Area of a great one, and is ſometimes called the little ſubtendent, and reere ſubtendent.*

Subtendentall, *is the ſubtendent of a great rectangled Triangle, within whoſe capacity is included a little one: it is likewiſe called the great ſubtendent, and maine ſubtendent.*

Supernumerary, *is ſaid of the digit, by the which the propoſed number exceeds in places the number of the places of the Radius.*

Supplements, *are the Oblongs made of the Segments of the root of a Square; and ſo called, becauſe they ſupply all that the Diagonals or*

*Squares*

*Squares of the Segments joyned together, want of the whole lines ſquare.*

Suppone ſeverally, *is to ſignifie ſeverall things.*

Suſtaining ſide, *is the ſubſterned, or ſubjacent ſide.*

Suſtentative, *is the ſame with ſuſtaining, ſubſterned, ſubjacent and Baſe.*

Sympathie *of Angles, is a ſimilitude in their affection,* of σὺν, and πάθος, paſſio, *vide* homogeneall.

## T.

TAble, *is an Index ſometimes, and ſometimes it is taken for a Briefe and ſummary way of expreſsing many things.*

Tangentine, *is that which concerneth Tangents or touch-lines.*

Tangentoſinall, *is ſaid of the Concordance of thoſe Loxogonoſphericals, the termes of whoſe Analogie runne upon Tangents and Sines.*

Tenet, *is a ſecondary maxime, and is onely ſaid here of Cathetothetick principles.*

Theorematick, *ſpeculative, from* θεώρημα, a ſpeculation, *which cometh from* θεωρεῖν, *or* θεωεῶν, ſpeculare, *or* contemplare.

Topanglet, *and verticalin are the ſame.*

Trigonometry, *is the Art of calculating, and meaſuring of Triangles, of* τρίγωνες, triangulus, *and* μετρέω, metior.

Triſſotetras, *is that which runneth all along upon threes and foures of* τριαδὸς, *and* in plurali, τρίανοι, tertius, trinus, triplex, tres, *and* τετρὰς, numerus quaternarius, *from* τέτταρες, quatuor.

## U.

VAriator, *is from* vario, variare, *to diverſifie, and is ſaid of caſes, which upon the ſame Datas are onely diverſe in the manner of reſolving the quæſitum.*

Verticaline, *verticall, verticalet, are the top-angles, and top-anglets, from* vertex, verticis.

Underproblem, *problemet, ſubordinate problem, ſub-problem, under-datoquære, and ſub. datoquære are, all the ſame thing.*

Unradiated, *or unradiuſed, is ſaid of a ſumme of Logarithms from which the Radius is abſtracted.*

## Z.

Zeterick, *is ſaid of Loxogonoſpherical Moods which agree in the ſame quæſitas, from* ζητέω, quæro, inquiro.

## *The finall Concluſion.*

IF the novelty of this my Invention be acceptable (as I doubt not but it will) to the moſt Learned, and judicious Mathematicians, I have already reaped all the benefit I expected by it, and ſhall hereafter, (God willing) without hope of any further recompence, cheerfully under-goe more laborious employments, of the like nature, to doe them ſervice: But as for ſuch, who, either underſtanding it not, or vain-gloriouſly being accuſtomed to Criticiſe on the Works of others, will preſume to carp therein at what they cannot amend, I pray God to illuminate their Judgements, and rectifie their Wils, that they may know more, and cenſure leſſe; for ſo by forbearing detraction, the venom whereof muſt needs reflect upon themſelves, they will come to approve better of the endeavours of thoſe, that wiſh them no harme.

*Sit Deo Gloria.*

## *The Diorthoſis.*

THe miſtakes of the Preſſe, can breed but little obſtruction to the progreſſe of the ingenious Reader, if with his Pen, before he enter upon the peruſall of this Treatiſe, he be pleaſed thus to correct (as I hope he will) theſe enſuing

### *Erratas.*

| *Pag.* | *lin.* | *Errata.* | *Emendata.* | *Pag.* | *lin.* | *Errata.* | *Emendata.* |
|---|---|---|---|---|---|---|---|
| 8 | 25 | Talfagro Talzo. | Talfagros Talzos. | 16 | 25 | This Cheme. | the Schemes |
| 10 | 17 | Niubprodeſver. | Niubprodneſver. | 16 | 31 | or dining roome | & dining room |
| 11 | 29 | Natfr. Autir. | Natfr. Eutir. | 23 | 23 | totall Sine. | totall ſumme. |
| 11 | 35 | Nat. Nad. Nath. | Næt. Næd. Næth. | 26 | 6 | as the Sine of | as the coſub- |
| 11 | 36 | Eheromabme. | Eherolabme. | | | the coſubten- | tendent. |
| 11 | 37 | Being Allotted. | Being abinarie allotted. | | | dent. | ſecond baſidi- |
| 16 | 8 | | | 80 | 9 | ſecond Baſe. | on. |

What errors elſe (if any) have ſlipt animadverſion (beſides their not being very materiall) are ſo intelligible, that being by the eaſieſt judgement with as much facility eſchewable, as I can obſerve them, not to mention the commiſſion of ſuch faults is no great omiſſion; and therefore will I heartily (without further ceremony) conduct the Student (who making this the beginning of the Book, as it is moſt fit he doe, ſeeing a Ruler ſhould be made ſtreight before any thing be ruled by it) is willing to go along with me from hence circularly through the title, to the end of the Treatiſe in the propoſed way, as followes.

*And ſo God bleſſe us both.*